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Monetary Policy Rules and the International Monetary Transmission

Leonor Coutinho*

Abstract

This paper analyses alternative monetary policy rules for the ECB, using a two "country" model of the euro area and the US, that assumes monopolistic competition, sticky prices and optimizing agents. The alternative rules analyzed for the ECB are ranked by their ability to stabilize consumption, output, and inflation and maximize consumers' welfare. The analysis contributes toward understanding the trade-offs faced by policymakers in open economies and provides some support for the current design of the ECB's operational framework. The results suggest that stabilizing money-growth, in addition to inflation, gives an additional degree of freedom to stabilize output. Although price stability is likely to remain the primary objective of the ECB, monetary policy must "without prejudice of price stability (...) support the general economic policies in the Community..." (Article 2). Hence monitoring money, under certain assumptions about the shocks hitting the economy, may deliver a better outcome in terms of output stabilization which should allow the ECB to fulfill its secondary but nonetheless important commitment.

JEL classification: F41, F42, E58, E31

Keywords: European Central Bank (ECB), monetary policy rules, stabilization, new open economy macroeconomics.

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1 Introduction

The Maastricht Treaty has firmly established that the "primary objective of the European Central Bank (ECB) shall be to maintain price stability" [but] "...without prejudice of price stability it shall support the general economic policies in the Community..." (Article 2). In order to achieve this objective the ECB has adopted a two pillar strategy consisting of, under the first-pillar, a reference value for money growth, and under the second-pillar a wide range analysis of economic and financial indicators that may contain information about prices. This strategy of the ECB has been criticized on the grounds that its parallel focus on monetary as well as price developments renders it opaque and confusing to market participants. Instead, the critics have argued, a policy focused on a single intermediate (either in the form of an inflation or money growth) target, under a unified framework or pillar, would improve the effectiveness and communication of monetary policy in the euro area.

From a theoretical viewpoint, the question of interest is whether such a framework can be welfare maximizing for the "representative" euro-area citizen. In order to address this issue this paper revisits a distinction, which dates back to Poole (1970), between "pure" (or single) target policies, and "combination" policies. A combination policy implies, for example, that the Central Bank targets a weighted average of two or more intermediate targets.[1] Based on this idea it examines whether, and under what conditions, a combination policy – perhaps as practiced by the ECB – may be welfare maximizing. The analysis is conducted within an open-economy model with utility-optimizing agents, sticky prices (or wages), and monopolistic competition (either in the product or the labor market). Price stickiness results in a sub-optimal, from a welfare maximization viewpoint, outcome and calls for more active macro stabilization policies (see Obstfeld and Rogoff, 1995).[2]

A simple open-economy model is developed comprising of the euro area and the US as the representative currency blocks. The ECB and the Federal Reserve System (FED) are responsible for monetary policy in the euro area, and the US, respectively. The model is similar, in spirit, to that of Corsetti and Pesenti (1999). However, instead of introducing money in the utility function this paper uses a cash-in-advance formulation in order to avoid having the nominal interest rate and the money growth rate changing proportionally to one another. This characteristic, which is a feature of the model by Corsetti and Pesenti (1999) model, would imply that the interest rate

[1]For simplicity, here as in Poole (1970) there will be no formal distinction between instruments and targets.

[2]Examples of recent open economy models that combine short-run rigidities with monopolistic competition are those by Obstfeld and Rogoff (1996), Corsetti and Pesenti (1999), Ghironi (1998), and Obstfeld and Rogoff (1998).

and monetary targeting alternatives are identical and hence it would not be desirable for the purposes of this paper (see also Sargent, 1987, on this point). In addition, it is assumed that, for credibility reasons, the ECB must commit to a pre-announced monetary policy rule while the FED, having acquired reputation among the market participants, may choose to "surprise" the market. Since, for simplicity, velocity is set equal to unity the only disturbances in the euro-area's monetary sector are, in effect, US money supply shocks. Other types of shocks, including fiscal shocks and shocks to preferences, are also analyzed although it is shown that these are less interesting cases.[3]

Three different policies – money growth targeting, interest rate targeting and inflation targeting – are compared in terms of their effectiveness in mitigating the external shock. These alternative policies are ranked according to their ability to stabilize inflation and, alternatively, the utility of consumers. Utility stabilization in this case will be the best that the monetary authority will be able to achieve in terms of maximizing consumers' welfare.[4]

Inflation targeting ranks (unsurprisingly) first in its ability to stabilize inflation. However, the ranking on the other two policies depends on the share of euro-area imports in total trade. The higher this share, the less attractive–for inflation stabilization purposes–interest rate targeting becomes (due to the stronger impact on the exchange rate) in comparison to money growth targeting.[5] However, in terms of utility stabilization the ranking of policies depends on the relative weight of consumption and leisure in the utility function. If the weight of consumption in the utility function is one (and the weight of leisure is zero), then it is shown that interest rate targeting is the best instrument to achieve utility stabilization. In contrast, if the representative agent values only leisure, a money growth targeting rule (which results in more output stabilization) is the preferred intermediate target for the purpose of stabilizing utility. However, in the intermediate case in which consumers value both consumption and leisure, a "combination" policy which takes into account the weight

[3]The cash-in advance formulation has the advantage of allowing for a closed form solution which, like in Corsetti and Pesenti (1999), facilitates the analysis of the impact of large disturbances and not only of small shocks, since it does not rely on log-linearization techniques. The assumption of constant velocity is also a simplification and could be relaxed following Woodford (1991).

[4]Shocks will be assumed to be normally distributed with mean zero. While "positive" shocks improve welfare, "negative" shocks reduce it. The expected mean of these variations will be zero but will be assumed that their variance has a cost. The policy loss function in this paper is in the same spirit as the Lucas welfare measure, used by Collard, Dellas and Ertz (2000), which consists of calculating how much consumption consumers would be willing to sacrifice to perfectly avoid any utility fluctuations.

[5]Uncovered interest parity holds in this model. Therefore, under a fixed interest rate regime, the exchange rate has to adjust fully to changes in the US interest rate.

of both consumption and leisure (output) in the utility function, can stabilize welfare optimally.[6]

Poole (1970) presented an analogous result for a central bank aiming at stabilizing output in a closed economy. Monetary targeting was found, in that case, to be more effective in stabilizing shocks in the real sector, while interest rate targeting better for stabilizing velocity shocks. He also showed that a "combination" policy - combining interest rate and money growth targeting by taking into account the variance of the different types of disturbances - could outperform the other policies.[7] In our open economy model, the results also suggest that, under certain conditions, a "combination strategy" can be welfare improving. Since the paper considers three possible targeting strategies, and only two targets are sufficient to stabilize consumers' utility, the "combination" policy can take the form of either money growth and interest rate targeting, inflation and interest rate targeting, or, inflation and money growth targeting. It is shown that, when policies are designed so as to produce the desirable outcome in terms of consumption and leisure (output) variations, all of these three choices can "insulate" welfare from the foreign money supply shocks.[8] However, in order to achieve this outcome, policymakers need to know the underlying parameters of the model (e.g., the degree of risk aversion, the market power of workers, the intertemporal discount rate, etc.) Consequently, when uncertainty is high regarding these parameters, a poorly designed "combination" policy can exacerbate the effect of shocks on consumption and output. Interestingly, a policy combining the inflation rate and the money growth rate does not require the knowledge of the parameter of risk aversion to guarantee that welfare is stabilized successfully.

The remainder of this paper is organized as follows: section two describes the model and the solutions; section three compares money growth targeting, interest rate targeting and inflation targeting in terms of inflation and welfare stabilization. Section four analyses the "combination" strategies. Section five outlines the effects of other policy alternatives and shocks and section six concludes.

[6] In an open economy domestic consumption is a share of aggregate world output supply. Therefore, for domestic consumption to remain stable foreign output changes must be offset by domestic output changes. This is the reason why there is a trade-off between keeping domestic output stable and keeping consumption stable, when the economy is open.

[7] Collard, Harris and Dellas (2000) generalize Poole results to the case of a general equilibrium model, still in a closed economy setup, and conclude that this policy trade-off disappears: nominal interest rate targeting results in greater stability than money growth targeting independently of the type of shock, generating also a higher level of welfare. In this analysis, however, the open economy set-up restores a policy trade-off in terms of utility stabilization.

[8] This paper will show that this is possible for all three types of combinations, by finding a relationship between the instruments which maintains utility stable at all times. For all three sets of instruments considered, the "optimal" combination policies yield the same outcome for inflation.

2 The Model

The world is comprised of two economies, Europe (EU) and the United States (US). As in Corsetti and Pesenti (1999), each economy in inhabited by an infinitely lived representative consumer, who decides how much to consume and work, and how many cash-balances and assets to hold under perfect foresight. Each economy specializes in the production of a specific basket of goods and the representative consumers derive utility from consuming both European goods and US goods. The utility-based consumption indexes of individual j in Europe, and individual j^*in the US, at time t, can be written as follows:

$$\begin{aligned} C^{EU}(j)_t &= C^{EU}_{eut}(j))^{\gamma}(C^{EU}_{ust}(j))^{1-\gamma} \\ C^{US}(j^*)_t &= C^{US}_{eut}(j^*))^{\gamma}(C^{US}_{ust}(j^*))^{1-\gamma} \end{aligned} \tag{1}$$

where C^{EU}_{eu} and C^{EU}_{us} are the amounts of European goods and US goods that are consumed by individual j, in Europe, while C^{US}_{eu} and C^{US}_{us} are the amounts of the home and the foreign goods, respectively, that are consumed by individual j^*, in the US. Starred parameters will refer to the US economy; subscripts denote the origin of the variables (or their currency denomination, in the case of prices and cash-balances), while superscripts denote the market where they are sold/transacted (or to which they apply, in the case of price deflators). The parameter γ can be interpreted as the share of the European goods in the world market, while $(1-\gamma)$ is the share of US goods. Assuming that there is no market segmentation across countries, so that the law-of-one-price holds, the consumption-based price indexes, P^{EU} and P^{US}, defined as the minimum expenditure needed for acquiring one unit of the composite goods C^{EU} and C^{US}given the price of the European basket P_{eu} denominated in euros, and the price of US basket P_{us} denominated in dollars are given by the following expressions:

$$\begin{aligned} P^{EU}_t &= \frac{1}{\gamma_w}(P_{eut})^{\gamma}(E_t P_{ust})^{1-\gamma} \\ P^{US}_t &= \frac{1}{\gamma_w}(P_{eut}/E_t)^{\gamma}(P_{ust})^{1-\gamma} \end{aligned} \tag{2}$$

where E_t is the exchange rate defined as the price of converting dollars into euros.[9] Notice that, because preferences are symmetric, the law-of-one-price implies that PPP holds between price deflators.

Firms in both economies operate under perfect competition. However, since labor inputs are differentiated and firms use a continuum of inputs in production, the labor

[9] The parameter γ_w is a function of the shares of European and US goods in the world market, such that $\gamma_w \equiv (\gamma)^{\gamma}(1-\gamma)^{1-\gamma}$.

market operates under monopolistic competition. In Europe, for example, producers set their production (Y_{eu}), given the price of the home product (P_{eu}), wages (W_{eu}), and the labor supply (L_{eu}), so as to maximize profits, subject to a production function with imperfect substitutability across labor inputs:

$$\begin{aligned} &\underset{L,Y}{Max}\left[P_{eut}Y_{eut}-\textstyle\int_0^1 W_{eut}(j)L_{eut}(j)dj\right] \\ &\qquad\qquad s.t.\quad Y_{eut}=\left[\textstyle\int_0^1 L_{eut}(j)^{\frac{\phi-1}{\phi}}dj\right]^{\frac{\phi}{\phi-1}} \qquad \phi>1 \end{aligned} \tag{3}$$

Notice that the larger is ϕ (the elasticity of substitution across inputs) the lower is the market power of workers. From the first order conditions it is possible to obtain an expression for the equilibrium total wage income of individual j, in Europe, and an analogous expression for the US:

$$W_{eut}(j)L_{eut}(j)=L_{eut}(j)^{1-\frac{1}{\phi}}P_{eut}Y_{eut}^{\frac{1}{\phi}}$$

Following Corsetti and Pesenti (1999), nominal rigidities are introduced in the short run in the form of predetermined nominal wages. Following a shock, wages can only adjust to their long-run steady state level after one period. In a symmetric equilibrium, in which all workers supply the same amount of labour, that is $L_{eut}(j)=L_{eut}$, for all j, it follows that $Y_{eut}=L_{eut}$ and $W_{eut}=P_{eut}$, from the production function and the first order condition, respectively. Hence the nominal wage rigidity is fully translated into a short-run price rigidity.

At the same time, consumers in Europe and in the US have to make decisions regarding their consumption level, their savings, and the labour supply. At the beginning of period τ, consumers can trade assets using their savings from the previous period and, after that financial exchange, they can buy goods using cash balances denominated in the currency of the producer. The savings can then be used in the next financial exchange. Given these assumptions, the representative European consumer, for example, maximizes the intertemporal utility U:

$$U=\sum_{\tau=0}^{\infty}\beta^{\tau}\left[\frac{C^{EU}(j)_{\tau}^{1-\rho}}{1-\rho}+V(G_{eu\tau})-\frac{k}{2}L_{eu}(j)_{\tau}^{2}\right] \qquad \rho>0,\rho\neq 1$$

subject to the cash-in-advance constraints (4) and to the financial constraint (5), where $M_{eu}^{EU}(j)_{\tau}$, $M_{us}^{EU}(j)_{\tau}$ stand for the nominal quantities of cash denominated in euros and dollars that are demanded by individual j in Europe.[10] In addition, $A_{eu}^{EU}(j)_{\tau}$

[10] The variables for US consumers can be obtained by replacing the superscript EU with US.

and $A_{us}^{EU}(j)_\tau$ are European and US real assets respectively; E_τ stands for the nominal exchange rate, and $\Omega_{\tau-1}$ is the wealth carried over from the previous period. Therefore:

$$C_{eu}^{EU}(j)_\tau \leq \frac{M_{eu}^{EU}(j)_\tau}{P_{eu\tau}} \qquad C_{us}^{EU}(j)_\tau \leq \frac{M_{us}^{EU}(j)_\tau}{P_{us\tau}} \tag{4}$$

$$M_{eu}^{EU}(j)_\tau + E_\tau M_{us}^{EU}(j)_\tau + P_\tau^{EU} A_{eu}^{EU}(j)_\tau + P_\tau^{EU} A_{us}^{EU}(j)_\tau \leq \Omega^{EU}(j)_{\tau-1} \tag{5}$$

The wealth $\Omega_{\tau-1}$consists of last period's wage income, lump-sum transfers, cash balances, real assets and interest earnings, accumulated over the previous period.[11] Due to arbitrage in the asset market, the real interest rates in the two economies have to equalize, so that $r_{eut} = r_{ust} = r_t$, for all t. An analogous problem is also solved by consumers in the US.

In each country, the governments can only finance their spending through taxes, or by issuing debt, while the central bank issues fiat money and distributes seigniorage revenues to the public in the form of lump-sum transfers.[12] Each period t, the nominal value of these lump-sum transfers in both countries must be equal to the increase in the nominal money stock, such that $P_t^{EU} TCB_{eut} = (M_{eut+1} - M_{eut})$ for the euro area, and $P_t^{US} TCB_{ust} = (M_{ust+1} - M_{ust})$ for the US, where TCB_{eut} and TCB_{ust} denote the ECB's and FED's real seigniorage transfers, respectively. Assuming that the governments collect lump-sum taxes TG_{eut} and TG_{ust}, the fiscal budget constraints can be written as:[13]

$$\begin{aligned} B_{eut+1} - B_{eut} &= r_{eut} B_{eut} + \frac{P_{eut} G_{eut}}{P_t^{EU}} - TG_{eut} \\ B_{ust+1} - B_{ust} &= r_{ust} B_{ust} + \frac{P_{ust} G_{ust}}{P_t^{US}} - TG_{ust} \end{aligned} \tag{6}$$

Consequently, the total net transfers received/paid by the consumers in Europe and in the US, which were previously denoted by T_{eut} and T_{ust}, consist of the difference between the corresponding seigniorage transfers and the corresponding lump-sum taxes, such that $T_{eut} = TCB_{eut} - TG_{eut}$ and $T_{ust} = TCB_{ust} - TG_{ust}$. The private budget constraints, together with the government budget constraints also imply that at each point in time the current accounts of each economy must satisfy the following

[11] The expression for Ω_{t-1} is in the appendix.

[12] The assumptions about the government budget constraints differ from those made by Corsetti and Pesenti (1998), who assume no government bonds and a consolidated budget constraint for monetary and fiscal policy. Instead, the assumptions here follow closely Ghironi's assumptions (Ghironi, 1998).

[13] For convenience, the European fiscal stances will be treated as an aggregate.

relationships:

$$NFA_t^{EU} = (1 + r_{eut-1})NFA_{t-1}^{EU} + \frac{P_{eut}(Y_{eut} - G_{eut})}{P_t^{EU}} - C_t^{EU} \tag{7}$$
$$NFA_t^{US} = (1 + r_{ust-1})NFA_{t-1}^{US} + \frac{P_{ust}(Y_{ust} - G_{ust})}{P_t^{US}} - C_t^{US}$$

where $NFA_t^{EU} = A_{eut}^{EU} + A_{ust}^{EU} - B_{eut}$ and $NFA_t^{US} = A_{eut}^{US} + A_{ust}^{US} - B_{ust}$ are the net foreign assets in Europe and in the US, respectively. In addition, equilibrium in asset markets requires that: $A_{eut}^{EU} + A_{eut}^{US} = B_{eut}$ and $A_{ust}^{EU} + A_{ust}^{US} = B_{ust}$. All these conditions imply that global net foreign assets expressed in a common currency must be zero, that is $P_t^{EU} NFA_t^{EU} + E_t P_t^{US} NFA_t^{US} = 0$, and given that PPP holds, this condition can be expressed as $NFA_t^{US} = -NFA_t^{EU}$.

Finally, both goods market are in equilibrium as long as demand equals supply. Therefore, equations (8) give the goods markets equilibrium conditions;

$$P_{eut}(Y_{eut} - G_{eut}) = P_{eut}\left(C_{eut}^{EU} + C_{eut}^{US}\right) \tag{8}$$
$$P_{ust}(Y_{ust} - G_{ust}) = P_{ust}\left(C_{ust}^{EU} + C_{ust}^{US}\right)$$

Similarly, the money market equilibrium, in which the supply of both home and foreign money meet their corresponding demands, is given by equations (9):

$$M_{eut} = M_{eut}^{EU} + M_{ust}^{US} \qquad M_{ust} = M_{ust}^{EU} + M_{ust}^{US} \tag{9}$$

Using these results and the restrictions imposed by the cash-in-advance constraints it is possible to obtain a quantity theory of money equation with velocity equal to one:

$$M_{eut} = P_{eut}C_{eut}^{EU} + P_{eut}C_{eut}^{US} \Rightarrow M_{eut} = P_{eut}(Y_{eut} - G_{eut}) \tag{10}$$
$$M_{ust} = P_{ust}C_{ust}^{EU} + P_{ust}C_{ust}^{US} \Rightarrow M_{ust} = P_{ust}(Y_{ust} - G_{ust})$$

The steady state equilibrium is defined as a sequence of prices and allocations in both countries for which: (i) consumption and output are constant; (ii) governments' spending and the money supplies do not change; (iii) cash-in advance constraints are binding; (iv) the consumers' budget constraint and the government budget constraint are satisfied with equality; (v) all first order conditions, including the Euler equation, hold with equality; (vi) markets clear and $B_t = \overline{B}, \forall_t$; and (vii) the transversality conditions are satisfied. From (i) and because the Euler equation must hold, it follows that in the steady state, $r = \frac{1-\beta}{\beta} \equiv \delta$. Since the money supply and government spending remain unchanged in the steady state and (i) holds, then it follows from equations (10) that prices will also remain constant and inflation will therefore be zero. In turn, zero inflation and a constant long-run real interest rate equal to δ, imply a constant long-run nominal interest rate also equal to δ.[14]

Given the preceding definition assume that at time $t = 0$ the economy is at its steady state, while at $t = 1$ there may be shocks to the money supply and/or to government spending. Since there is perfect foresight while wages take one period to adjust, the problem then is reduced to a two period model, with a new steady state in the second period. For simplicity, let variables with a bar denote steady state equilibrium values. The initial steady state can be distinguished from the final steady state by a time zero subscript.

The solution of the model is presented in Appendix A.2. It can be obtained by noticing that the current account equations provide a link between the short-run and the long-run. Assuming that the initial level of debt is equal to zero, the equilibrium in the external sector implies that there is no international lending or borrowing, and that the ratio between the European and the US consumption is equal to the relative market shares of European and US products in the world market, both in the long and in the short-run. The result that policy shocks have no impact on the current accounts stems from the assumption of a unitary intratemporal elasticity of substitution between the composite goods produced in each country. In this model, just as in Corsetti and Pesenti (1999) and in Obstfeld and Rogoff (1998), when the income of the consumers of one country increases relatively to the income of consumers abroad, their purchasing power declines proportionally, due to the Cobb-Douglas specification of preferences, leaving no incentives for international lending or borrowing.[15] This result together with the first order conditions allow for three equilibrium loci in this model, YC_1, YC_2 and MC, which can be found in Appendix A.2. The locus YC_1 is obtained from the goods market equilibrium and implies a relationship between consumption and output; the locus YC_2, derived from the labour-leisure trade-off first order condition, also provides a relationship between consumption and output; and the locus MC, derived from the cash-in-advance constraints and the quantity theory equation, produces a relationship between consumption and real money balances.

In the long run, both the labor-leisure trade off and the goods market equilibrium will be binding in the two economies. These two relations determine the two equilibrium loci between consumption and output, YC_1 and YC_2, which are enough to

[14] The first order conditions are shown in Appendix A.1

[15] The fact that price responses can make international trade in securities redundant with Cobb-Douglas preferences was pointed out by Cole and Obstfeld (1991).For more details about the intuition for this result see Corsetti and Pesenti (1999). Obstfeld and Rogoff (1995) and Ghironi (1998) obtain a role for the current account in their model specifications, but at the expense of analytical tractability. In their models, the elasticity of intratemporal substitution is different from one and is the same as the index of monopolistic distortion, because their monopolistic competition arises from assumptions on preferences.

determine the long-run levels for these variables in Europe and in the US. In the long run, money will have no effect on consumption and output and it will only determine prices. Money will not have an effect on the terms of trade either:

$$\overline{C}^{EU} = a_c \beta^{\frac{1}{1+\rho}} (\overline{g}_w)^{-\frac{1}{1+\rho}} \tag{11}$$

$$\overline{Y}_{eu} = a_y \beta^{\frac{1}{1+\rho}} (\overline{g})^{\frac{1}{2}} (\overline{g}_w)^{-\frac{1-\rho}{2(1+\rho)}} \tag{12}$$

$$\overline{P}^{EU} = (a_c)^{-1} \beta^{-\frac{1}{1+\rho}} \overline{g}_w^{\frac{1}{1+\rho}} \overline{M}_{eu} \tag{13}$$

$$\overline{P}_{eu} = (a_c)^{\rho-1} a_y \Phi^{-1} \beta^{-\frac{1}{1+\rho}} (\overline{g}_{eu})^{\frac{1}{2}} (\overline{g}_w)^{\frac{1-\rho}{2(1+\rho)}} \overline{M}_{eu} \tag{14}$$

$$\overline{E} = \frac{1-\gamma}{\gamma} \overline{M}_R \tag{15}$$

$$\frac{\overline{EP}_{us}}{\overline{P}_{eu}} = a_p (\overline{g}_R)^{-\frac{1}{2}} \tag{16}$$

where a_c, a_y and a_p are parameters defined in Appendix A.2.[16] Analogous solutions will hold for the US.

In the short run, on the other hand, in the presence of nominal rigidities and monopolistic competition in the labor market, agents do not necessarily operate on their labor supply schedule, so that the locus YC_2, defined by the leisure labour trade-offs, does not determine the short-run solution.[17] During the period when prices remain fixed, consumption and output levels are simply determined by the cash-in-advance constraints and the goods market equilibrium, and therefore by the

[16] As it is shown in the appendix, $g_{eu} \equiv Y_{eu}/(Y_{eu} - G_{eu})$ and $g_{us} \equiv Y_{us}/(Y_{us} - G_{us})$. In addition the following world aggregates (x_{wt}) and relative indexes (x_{Rt}) where also defined, such that $x_{wt} \equiv (x_{eut})^{\gamma}(x_{ust})^{1-\gamma}$ and $x_{Rt} \equiv x_{eut}/x_{ust}$, where x is the relevant variable. Similar aggregated are also used for parameters (see the appendix).

The long-run equilibrium of this model is almost analogous to the one of the model set up by Corsetti and Pesenti (1998). The only difference is the factor $\beta^{\frac{1}{1+\rho}}$, which appear in this version due to the fact that the cash-in-advance requirements make labor income only available with a period of lag.

[17] It will be assumed that the range of shocks is such that the real wage does not fall below the marginal rate of substitution between consumption and leisure, otherwise workers would not be willing to modify their work effort at a fixed nominal wage. See Corsetti and Pesenti (1999) for more details.

equilibrium loci YC_1 and MC.[18]

$$Y_{eu} = \overline{Y}_{eu0} \frac{M_{eu}}{\overline{M}_{eu0}} \frac{g_{eu}}{\overline{g}_{eu0}} \tag{17}$$

$$C^{EU} = \overline{C}_0^{EU} \frac{M_w}{\overline{M}_{w0}} \tag{18}$$

$$P^{EU} = (a_c)^{-1} \beta^{-\frac{1}{1+\rho}} (\overline{g}_{w0})^{\frac{1}{1+\rho}} \overline{M}_{w0} (M_R)^{1-\gamma} \tag{19}$$

$$P_{eu} = (a_c)^{\rho-1} a_y \Phi^{-1} \beta^{-\frac{1}{1+\rho}} (\overline{g}_{eu0})^{\frac{1}{2}} (\overline{g}_{w0})^{\frac{1-\rho}{2(1+\rho)}} \overline{M}_{eu0} \tag{20}$$

$$E = \frac{1-\gamma}{\gamma} M_R \tag{21}$$

$$\frac{EP_{us}}{P_{eu}} = a_p (\overline{g}_{R0})^{-\frac{1}{2} \frac{M_R}{\overline{M}_{R0}}} \tag{22}$$

Consumption in the short-run changes relative to its initial level proportionally to changes in the world money supply, because consumers must hold both currencies in order to be able to consume European and US goods. Since in this model goods must always be acquired with the currency of the producer, European output does not depend on the stock of dollars, but simply on the stock of euros.[19]

3 Alternative Monetary Policy Rules

How will a monetary policy shock, originating in the US, be transmitted to the European economy? Which kind of monetary policy rule may best insulate the European economy from foreign monetary policy disturbances? Assume there are three alternative rules: to target the interest rate, to target a constant rate of money growth, or set an inflation target. Fulfilling the model equilibrium conditions determines that not all targets can be set independently. Note in particular that the following relationships between the interest rate and money growth, and between money growth and inflation must hold:[20]

$$1 + i_{eu} = \frac{1}{\beta} \left(\frac{\overline{g}_w}{\overline{g}_{w0}} \right)^{\frac{1-\rho}{1+\rho}} \left(\frac{M_w}{\overline{M}_{w0}} \right)^{1-\rho} \frac{\overline{M}_{eu}}{M_{eu}} \tag{23}$$

[18] $\overline{C}_0^{EU} = a_c \beta^{\frac{1}{1+\rho}} (\overline{g}_{w0})^{-\frac{1}{1+\rho}}$ and $\overline{Y}_{eu0} = a_5 \beta^{\frac{1}{1+\rho}} \overline{g}_0^{\frac{1}{2}} (\overline{g}_{w0})^{-\frac{1-\rho}{2(1+\rho)}}$ are the initial steady-state equilibrium levels of the European consumption and output, respectively.

[19] Solutions in all respects analogous to these can be found for the case of the US, but will be omitted here for simplicity of exposition.

[20] Notice that an economy's interest rate is inversely related to the demand for cash balances denominated in its currency for any $\rho > 0$ and any $\gamma \in [0, 1]$.

$$\frac{P^{EU}}{\overline{P}_0^{EU}} = \left(\frac{M_R}{\overline{M}_{R0}}\right)^{1-\gamma} \tag{24}$$

$$\frac{\overline{P}^{EU}}{P^{EU}} = \left(\frac{\overline{g}_w}{\overline{g}_{w0}}\right)^{\frac{1}{1+\rho}} \frac{M_w}{\overline{M}_{w0}} \frac{\overline{M}_{eu}}{M_{eu}} \tag{25}$$

Assume now that there is only a permanent shock to the US money supply, such that $M_{us} \neq \overline{M}_{us0}$, but $\mu^* = \overline{M}_{us}/M_{us} = 1$; and that government expenditures remain fixed at g_{eu0} and g^*_{eu0}, both in the short and in the long run. Since monetary policy has no effect in the long run, output and consumption in the new steady-state are the same under both monetary policy rules and are equal to their initial levels.

3.1 Money Targeting

In this regime, the monetary authority commits to constant gross rate of money growth μ in the short run, and $\overline{\mu}$ in the long run, given the initial money stock $\overline{M}_{eu0}$. For simplicity it will be assumed that $\mu = \overline{\mu} = 1$, hence under this regime $M_{eu} = \overline{M}_{eu} = M_{eu0}$. The nominal interest rate is endogenously determined according to equation (23), such that:

$$1 + i_{eu} = \frac{1}{\beta}\left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{(1-\gamma)(1-\rho)} \tag{26}$$

Since the stock of euros is constant, output will also remain equal to its initial steady-state level, provided that fiscal policy remains unchanged (equation 18). Nonetheless a change in the US money supply will induce a change in the world money supply, which will in turn affect consumption, since:

$$\frac{M_w}{\overline{M}_{w0}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{1-\gamma} \tag{27}$$

In the short run, the exchange, which is determined by the relative short-run money supply M_R, will change proportionally to the money growth in the US, and, given the producer price rigidities, the consumer price index will simply follow the movement in the exchange-rate:

$$\frac{E}{\overline{E}_0} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{-1} \qquad \frac{P^{EU}}{\overline{P}_0^{EU}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{-(1-\gamma)} \tag{28}$$

In the long run, however, because the shock to the US money supply is assumed to be permanent, there will be no further adjustments in the exchange rate. In this case, there is an immediate adjustment of the exchange rate to its long-run level, that is

$E = \overline{E}$. The long-run effect of the US money shock on inflation can be found by substituting (27) on (25), and letting $\overline{M}_{eu} = M_{eu}$ and $\overline{g}_w = \overline{g}_{w0}$. In this case, the long-run price index movement offsets the short-run effect. The price level moves by the same amount in both periods but in opposite directions. When all adjustments take place, price deflators go back to their initial steady-state levels, in this case.

$$\frac{\overline{P}^{EU}}{P^{EU}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{1-\gamma} \tag{29}$$

3.2 Interest Rate Targeting

Under this policy regime, it is assumed that the monetary authority commits to fixing the nominal interest rate at its steady state equilibrium level, such that $i_{eu} = \delta$. Under this policy it is necessary to assume that expectations about the long-run money supply are known, otherwise there is an indeterminacy problem when the government simply targets the interest rate. Therefore, it will be assumed that $\overline{\mu} = \overline{M}_{eu}/M_{eu} = 1$, with M_{eu0} given. Substituting this policy rule into equation (23), allows us to derive the rate of growth of money in the euro area as a function of the US money supply:

$$\frac{M_{eu}}{\overline{M}_{eu0}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{-\frac{1-\gamma}{\gamma}} \tag{30}$$

In the short run, output will change proportionally to the change in the US money supply given that the domestic money stock will also change (equation 18). Consumption, on the other hand, will be insulated from US monetary shocks, since keeping the nominal interest rate fixed also stabilizes the world money supply. This occurs because UIP holds in this model, and because shocks are assumed to be permanent, such that $\overline{\mu} = \overline{\mu}^* = 1$, making the nominal interest rate respond only to changes in the world money supply.[21]

$$\frac{M_w}{\overline{M}_{w0}} = 1 \tag{31}$$

[21] In order to see this, notice that: $1+i_{eu} = (1+i_{us})\frac{\overline{E}}{E} \Rightarrow \frac{1+i_{eu}}{1+i_{us}} = \frac{\overline{M}_R}{M_R} \Rightarrow \frac{1+i_{eu}}{1+i_{us}} = \frac{\overline{\mu}}{\overline{\mu}^*}$,where $1+i_{us}$ can be replaced by the foreign counterpart of equation (23):$\frac{1+i_{eu}}{\beta^{-1}\overline{\mu}^*(M_w/M_{wo})^{1-\rho}} = \frac{\overline{\mu}}{\overline{\mu}^*}$. Hence: $1+i_{eu} = \beta^{-1}\overline{\mu}(M_w/M_{wo})^{1-\rho}$, when $\overline{\mu} = \overline{\mu}^* = 1$. When $\overline{\mu} \neq 1$, the same effect of keeping the world money demand constant would be produced by fixing $1+i_{eu} = \delta + \beta^{-1}(\overline{\mu}-1)$. When $\overline{\mu} > 1$, there is a pressure for the interest rate to go up through expected inflation, therefore, the interest rate must be higher than δ in order to offset this effect. When $\overline{\mu} < 1$ the opposite occurs.

Under this policy regime, the impact of a foreign money supply shock on the exchange rate, will depend on the world market share of European goods. The smaller this share, that is the larger γ, the higher the share of US goods in the world market and the higher the elasticity of the exchange rate to US money supply shocks. The price level will be determined by the changes in the nominal exchange rate:

$$\frac{E}{\overline{E}_0} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{-\frac{1}{\gamma}} \qquad \frac{P^{EU}}{\overline{P}_0^{EU}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{-\frac{1-\gamma}{\gamma}}$$

However, in the long-run there will be no further adjustments in the exchange rate ($\overline{E} = E$) and no further adjustments in the price indexes (since it was assumed that $\overline{\mu} = \overline{M}_{eu}/M_{eu} = 1$ in order to avoid the price level indeterminacy mentioned earlier). In this case, $\overline{P}^{EU} = P^{EU}$ will be above (below) the initial steady state level given the contractionary (expansionary) US monetary shock.

3.3 Inflation Targeting

Under an inflation targeting regime the central bank will aim at maintaining a constant rate of inflation π_0. Without changes in fiscal policy, this requires that:

$$\frac{P^{EU}}{\overline{P}_0^{EU}} = \left(\frac{M_R}{\overline{M}_{R0}}\right)^{1-\gamma} = 1 + \pi_0$$
$$\frac{\overline{P}^{EU}}{P^{EU}} = \frac{M_w}{\overline{M}_{w0}} \frac{\overline{M}_{eu}}{M_{eu}} = 1 + \pi_0$$

Assuming for simplicity that $\pi_0 = 0$, this implies that the European monetary authority must set the money supply according to the following reaction function:

$$\frac{M_{eu}}{\overline{M}_{eu0}} = \frac{M_{us}}{\overline{M}_{us0}}$$
$$\frac{\overline{M}_{eu}}{M_{eu}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{-1}$$

Notice that maintaining the price index constant is, in the short run, equivalent to fixing the nominal exchange rate, since output prices are fixed for one period. Therefore, under this policy regime, the short-run nominal exchange rate must remain at its initial level. In the long-run, although the real variables will not be affected by monetary policy, the nominal exchange rate may have to adjust depending on whether the US money shock is temporary (that is, is reversed after one period), or permanent.[22]

Notice that inflation targeting ranks (unsurprisingly) first in its ability to stabilize inflation. Since there is no uncertainty in the transmission mechanism the monetary authority can perfectly achieve price stability with this rule, while under the other two rules prices have to adjust to the US money supply shock in the long run. The ranking on the other two policies depends on the export share of European producers in the world market. The lower the share of European goods in the world market, ceteris paribus, the less attractive interest rate targeting becomes in comparison to money growth targeting for stabilizing inflation.[23]

3.4 Effects on Consumption and Output

Table 1 summarizes the impact on European consumption and European output of a shock in the US money supply, under the alternative policy scenarios. Since the utility of consumers depends on consumption and leisure (which is a function of output), these effects will be important in determining the trade-offs between one rule and another. As seen before, when the stock of euro money balances is kept constant, output suffers no changes but European consumption is affected by the shock. On the other hand, if the nominal interest rate is kept fixed, Eurozone consumption remains stable but output adjusts. Under inflation targeting, both consumption and output will respond to the US monetary shock, given the change in the European money supply needed to maintain the inflation target. Under this policy regime, the proportional increase in consumption is exactly equal to the proportional increase in output. Given these results, it is clear that the relative impact of each of these policies on the utility of consumers will depend on the relative weight of consumption and output in the utility function.

[22] In the long run, the nominal exchange rate must respond to short-run and long-run changes in the US money supply as follows:

$$\frac{\overline{E}^{EU}}{E^{EU}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\frac{\overline{M}_{us}}{M_{us}}\right)^{-1}$$

If the US monetary shock is reversed such that $\overline{M}_{us} = M_{us0}$, the nominal exchange rate $\overline{E}^{EU}$ will remain at the initial steady state level. However, if the increase in the US money supply is permanent, then $\overline{M}_{us} = M_{us}$ and the long-run nominal exchange rate will have to adjust by $\frac{M_{us}}{\overline{M}_{us0}}$.

[23] This result is shown in Appendix (A.3).

Table 1: Effects of a US money supply shock in Europe under alternative policy rules.

POLICIES / EFFECTS	$\overline{M}_{eu}$	$\bar{i}_{eu}$	$\overline{\pi}_{eu}$
ΔC^{EU}	$\left[\left(\frac{M_{us}}{\overline{\overline{M}}_{us0}}\right)^{1-\gamma}-1\right]C_0^{EU}$	0	$\left[\left(\frac{M_{us}}{\overline{\overline{M}}_{us0}}\right)-1\right]C_0^{EU}$
ΔY_{eu}	0	$\left[\left(\frac{M_{us}}{\overline{\overline{M}}_{us0}}\right)^{-\frac{1-\gamma}{\gamma}}-1\right]Y_{eu0}$	$\left[\left(\frac{M_{us}}{\overline{\overline{M}}_{us0}}\right)-1\right]Y_{eu0}$

$\Delta C^{EU} = C^{EU}$ ñ C_0^{EU} ; $\Delta Y_{eu} = Y_{eu}$ ñ Y_{eu0}

4 Policies for the ECB

The ranking of alternative policies available to the ECB will be based on each policy's ability to minimize the impact of exogenous shocks relative to the steady-state utility level. Formally the problem is to minimize the quadratic loss function:[24]

$$L_{eu} = E[(U - U_0)^2]$$

Since utility depends on both consumption and output, policies should take into account of changes in both these variables. Although targeting the money supply mitigates the effects of a foreign monetary shock on output, such a policy cannot stabilize consumption - which depends on the world money supply - fully. Conversely, a fixed nominal interest rate policy is successful at insulating consumption from the monetary policy shock, but not as a buffer to output changes. Hence the use of one of these policy rules in isolation cannot fully insulate welfare from an external foreign policy shock.

[24]This chapter shows that the policy which maximizes utility given a positive shock will be the one which minimizes it given a negative shock and vice-versa. Therefore the best rule to maximize utility at all times would be a switching rule, a type of "combination" rule which will not be considered here. If the expected value of shocks is zero, the best that monetary policy can do in terms of welfare under a non-switching rule type of regime is to insulate utility from shocks. this loss function is also in line with the Lucas welfare measure, used by Collard, Dellas and Ertz (2000) as one measure to evaluate their alternative policies. Such a measure consists of calculating how much consumption consumers would be willing to sacrifice to perfectly avoid any utility fluctuations.

In Poole's model (Poole, 1970), one policy was more successful than the other in stabilizing the economy depending on whether the shocks hitting the economy came from the expenditure sector, or from the monetary sector. To control for both types of shocks simultaneously, it was suggested that the monetary authority should pursue a combination policy, of a money and an interest rate target, in such a way that the weights take into account of the relative variance of the shocks. A similar trade-off between stabilizing the different components of utility (consumption and leisure) exits here, which can be addressed by a combination policy in a similar way to Poole's suggestion. In order to analyze this formally, it is convenient to rewrite the loss function, such that:

$$L_{eu} = \beta^2 E\left[\left(\left(C_0^{EU}\right)^{1-\rho}\ln\left(\frac{M_w}{M_{w0}}\right) - K\left(Y_{eu0}\right)^2\ln\left(\frac{M_{eu}}{M_{eu0}}\right)\right)^2\right] \tag{32}$$

using $x \simeq \ln(x+1)$.[25] Under a fixed monetary growth policy the loss function takes the following form:

$$L_M|_{eu} = \beta^2\left(1-\gamma\right)^2\left(C_0^{EU}\right)^{2(1-\rho)}\sigma^2_{M_{us}} \tag{33}$$

where $\sigma^2_{M_{us}}$ is the expected variance of the US monetary shock .Alternatively, if the ECB pursues a fixed nominal interest rate rule, the loss function will be given by:

$$L_i|_{eu} = \beta^2\left(1-\gamma\right)^2\left(\frac{1}{\gamma}K\left(Y_{eu0}\right)^2\right)^2\sigma^2_{M_{us}}$$

The two expressions can then be conveniently compared, by dividing one by the other. In addition define $\xi_{eu} \equiv \left((\overline{C}_0^{EU})^{(1-\rho)}\right) / \left(k(\overline{Y}_{eu0})^2\right)$, which can be understood as the relative weight of consumption versus output in the steady state utility function, to obtain:

$$\frac{L_M}{L_i}|_{eu} = \left(\gamma\xi_{eu}\right)^2 \tag{34}$$

where

$$\xi_{eu} = \frac{(\overline{C}_0^{EU})^{(1-\rho)}}{k(\overline{Y}_{eu0})^2} = \frac{\phi}{(\phi-1)}\frac{1}{\beta(\overline{g}_{eu0})}$$

Notice that ξ_{eu} is always positive.[26] Since ξ_{eu} and γ are positive, the ratio (34) is higher (lower) than one depending on whether ξ_{eu} is higher (lower) than the inverse of γ. Recall that γ represents the effective macroeconomic size of the European economy,

[25] The algebra needed to derive the results in this section is shown in Appendix A.4.

[26] The relative weight of consumption and output in the utility function, ξ_{eu}, will be higher, the higher the market power of workers (the lower ϕ), the lower the discount factor, and the lower the steady-state government spending G_{eu0} (the lower $\overline{g}_{eu0}$).

in terms of the weight of the European goods in the world consumption, therefore, the inverse of γ is proportional to the size of the US.

$$\xi_{eu} < \frac{1}{\gamma} \Rightarrow L_M|_{eu} < L_i|_{eu}$$
$$\xi_{eu} > \frac{1}{\gamma} \Rightarrow L_i|_{eu} < L_M|_{eu}$$

A fixed interest rate policy seems, therefore, to be more attractive for the European economy if the representative consumer attaches a higher weight on the utility derived from consumption rather than on the disutility from work. Conversely if the representative consumer attaches a larger weight on the disutility from work, it should be more prone to support a fixed money growth target. To illustrate graphically the choice between the nominal interest rate and the money growth rate for monetary policy targets two examples have been estimated, setting $g_0 = 1/0.9$, $\beta = 0.9$, $\rho = 4$, $\gamma = 0.5$ in both cases and different values of the parameter of monopolistic competition for each case. Figure 1 shows the case in which ϕ is high enough, given the assumptions on the remaining parameters (low market power of workers), so that $\xi < (1/\gamma)$. Figure 2 shows the case in which ϕ is low enough, given the values chosen for the other parameters (high market power of workers), such that $\xi > (1/\gamma)$. Here the actual utility function is plotted, rather than the loglinear approximation.

Figure 1: Utility Comparisons: $\xi < \dfrac{1}{\gamma}$

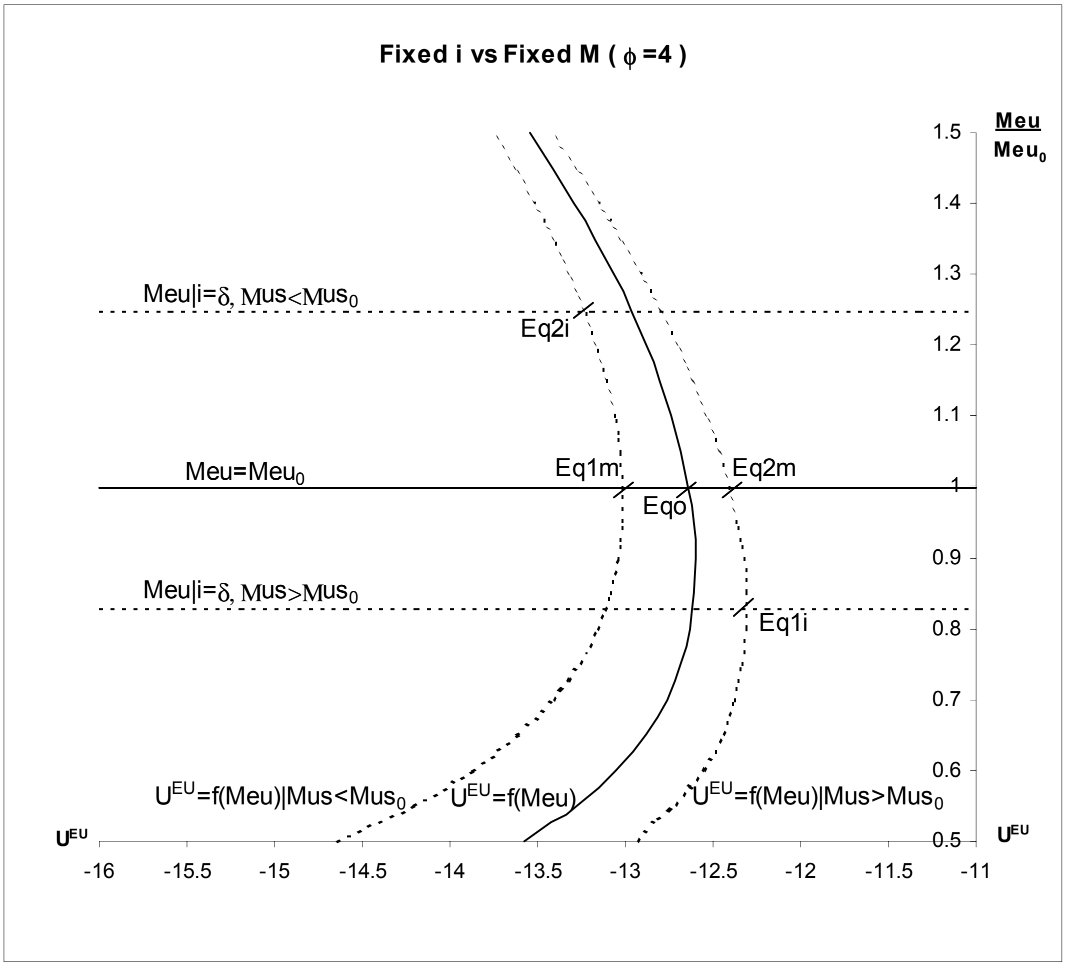
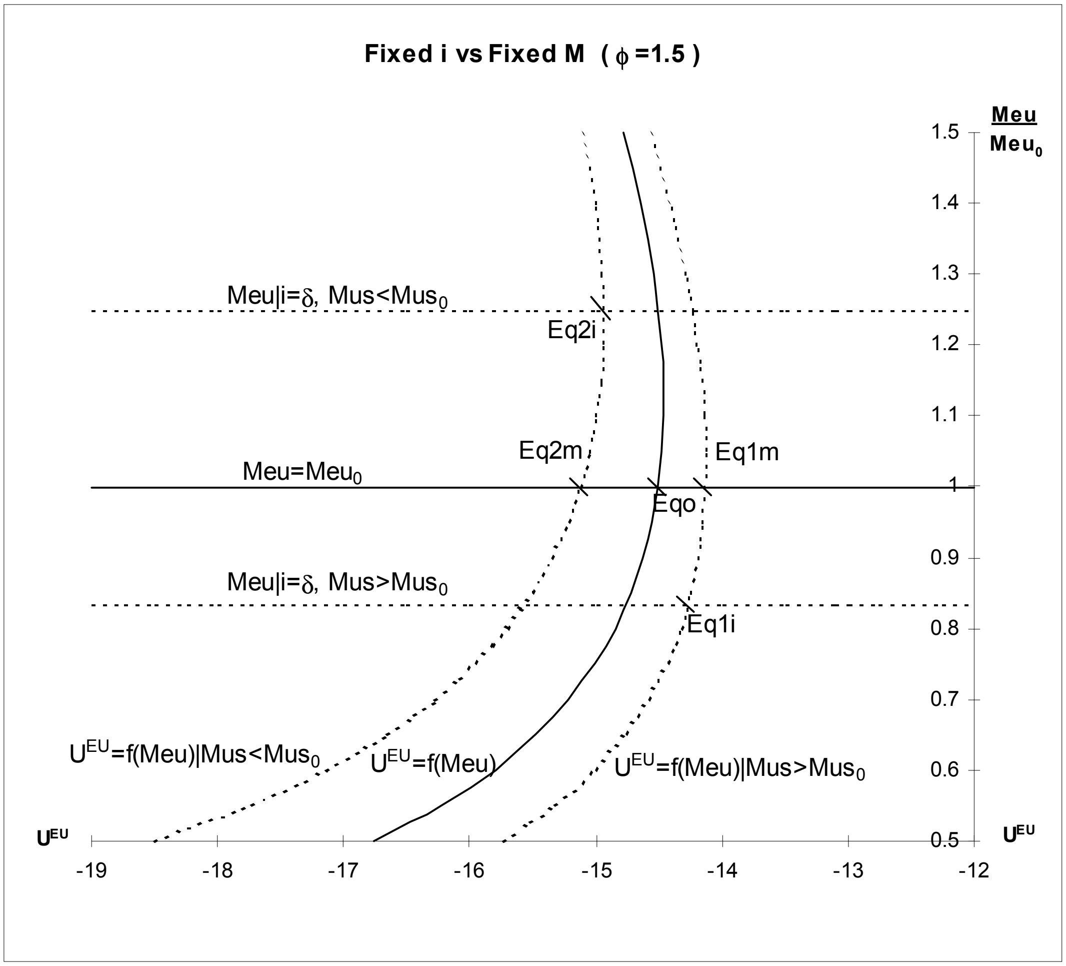


Figure 2: Utility Comparisons: $\xi > \dfrac{1}{\gamma}$



In both figures, the initial equilibrium point is denoted '*Eqo*'. The alternative final equilibrium points, depending on whether the US follows an expansionary or a contractionary monetary policy, given that Europe follows a constant money supply rule, are labeled '*Eq*1*m*' and '*Eq*2*m*', respectively. In Figure 1, these are closer, in utility terms, to '*Eqo*' than '*Eq*1*i*' and '*Eq*2*i*', which are the corresponding final equilibrium points when Europe follows a constant interest rate rule. In the case represented in Figure 1, where $\xi < (1/\gamma)$, the fixed money supply rule stabilizes utility more. In Figure 2, when $\xi > (1/\gamma)$, the opposite occurs: utility varies less when Europe follows a constant nominal interest rate rule.

From the previous analysis it can be seen that it is impossible to insulate completely the economy from foreign shocks using either instrument alone. This is because with one instrument you can only pursue one objective, while stabilizing the utility function entails two. However, a combination policy, of the family proposed by Poole, in which the money stock and the nominal interest rate are maintained in a certain relationship to each other can solve the problem. In this case, the optimal relationship will have to take into account of the relative weight of consumption and output in the utility function and the relative size of the countries. Formally, such a combination policy can be defined as a relationship of the type:

$$\left(\frac{M_{eu}}{M_{eu0}}\right)^{\alpha} = \left(\frac{1+i}{1+\delta}\right)^{1-\alpha}$$

which minimizes the loss function, and where α determines the relationship maintained between the money growth and the nominal interest rate target. The combination policy becomes a pure money growth rate policy, when $\alpha = 1$, or a pure interest rate policy when $\alpha = 0$. The value of the parameter α for which the loss function is zero for any value of $\sigma^2_{M_{us}}$, determines the monetary policy rule which insulates the economy from the US monetary shock. In this case that value is given by:[27]

$$\alpha = \frac{(\rho - 1)}{(\rho - 1) - \xi_{eu}}; \quad \xi_{eu} \neq \frac{1}{\gamma}$$

where ξ_{eu}, is defined as before in (34).

The inability to stabilize welfare with only one instrument can also emerge under inflation targeting. In this model, under inflation targeting, the foreign monetary disturbance affects equally consumption and output. Hence this policy will only stabilize utility fully if the weights of consumption and output/leisure in the utility function are the same. Otherwise, a combination policy including inflation and either the money stock growth or the nominal interest rate can do better. Below these two types of policies will be denoted *two-pillar policy with the nominal interest rate* and *two-pillar policy with the money growth rate.*

Formally, the *two-pillar policy with the nominal interest rate*, combining an inflation and a nominal interest rate target, can be described by the following relationship:

$$\left(\frac{P^{EU}}{P_0^{EU}}\right)^{\alpha_{\pi i}} = \left(\frac{1+i_{eu}}{1+\delta}\right)^{1-\alpha_{\pi i}}$$

which must minimize the loss function, and where $\alpha_{\pi i}$ determines the nature of the relationship between the variables in the combination. When $\alpha_{\pi i} = 1$ the combination policy is a pure inflation targeting policy. Conversely, when $\alpha_{\pi i} = 0$, it becomes a

[27] The calculations are shown in Appendix A.5.

pure fixed interest rate rule. Under this rule, the value of $\alpha_{\pi i}$ which allows to insulate the economy from the US monetary disturbance is given by:[28]

$$\alpha_{\pi i} = \frac{\rho - 1}{\rho - \xi_{eu}}; \quad \xi_{eu} \neq \frac{1}{\gamma}$$

If, on the other hand, the ECB decides to rely targeting the money stock, instead of the nominal interest rate, it will follow the *two-pillar policy with the money growth rate*. This strategy can be defined as a relationship between inflation and the money growth rate of the form:

$$\left(\frac{P^{EU}}{\overline{P}_0^{EU}}\right)^{\alpha_{\pi M}} = \left(\frac{M_{eu}}{M_{eu0}}\right)^{1-\alpha_{\pi M}}$$

which minimizes the loss function, and where $\alpha_{\pi M}$ determines the nature of the relationship between the two variables. This policy becomes a pure inflation target when $\alpha_{\pi M} = 1$, and a pure constant money supply policy when $\alpha_{\pi M} = 0$. In this case, the value of $\alpha_{\pi M}$ which sets the loss function equal to zero can be written as:[29]

$$\alpha_{\pi M} = \frac{\xi_{eu}}{2\xi_{eu} - 1}; \quad \xi_{eu} \neq \frac{1}{\gamma}$$

In all cases the optimal combination policy is superior compared with the single or "pure" policy rules.[30] It seems difficult to give preference for one kind of policy index over the other, since they all facilitate insulation of the economy from the US money stock disturbance. However, the simpler the rule, the easier it will be to implement it properly. As in Poole (1970), the success of the combination policy depends on knowledge of the parameters and even the sign of the optimal relationship cannot be determined without such a knowledge: a combination policy based on intuition can be worse than either of the pure policy rules. Within this framework, the fact that the *two-pillar policy with the money growth rate* (which combines the money stock and inflation) does not require knowledge of the parameter of risk aversion, seems to be an advantage. Nevertheless, all combination policies require the knowledge of the parameter of monopolistic competition. The parameter ϕ determines the kind of relationship to be maintained between the policy variables, for a given market size, γ, a given coefficient of time preference and a given steady state fiscal intervention parameter, $\overline{g}_{eu0}$.[31]

[28] The calculations are shown in Appendix A.5.

[29] See Appendix A.5.

[30] Except in the particular case where the relative weights of consumption and leisure in the utility function are the same. In that case inflation targeting can also achieve full welfare stabilization, but only for very restrictive set of parameters.

[31] Notice that it is only the level of the parameter of monopolistic competition in Europe that matters for the ECB policy decisions and not its size relatively to the size of the corresponding US parameter. This version of the model cannot exploit the impact of the differences between US and European labor markets.

It is important to notice though that all the "combinations" discussed so far allow the US money supply shock to pass-through to inflation, by the same amount. The changes in prices in the short run and in the long run are given by equations (35) and (36) whatever the "combination" policy pursued, when the policy parameter α_j is set according to the optimal welfare criteria discussed before, for $\xi_{eu} \neq 1/\gamma$:

$$\frac{P^{EU}}{\overline{P}_0^{EU}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{\frac{(1-\gamma)(\xi_{eu}-1)}{1-\gamma\xi_{eu}}} \tag{35}$$

$$\frac{\overline{P}^{EU}}{P^{EU}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{\frac{1-\gamma}{1-\gamma\xi_{eu}}} \tag{36}$$

The total change in the price level, resulting from the shock is therefore:

$$\frac{\overline{P}^{EU}}{\overline{P}_0^{EU}} = \left(\frac{M_{us}}{\overline{M}_{us0}}\right)^{\frac{(1-\gamma)\xi_{eu}}{1-\gamma\xi_{eu}}}$$

The only policy that allows to insulate inflation from the foreign money supply shock, in this model, is simple inflation targeting.

5 Other Policy Alternatives and Shocks

5.1 Other Policy Alternatives

The exchange rate is an alternative tool for the conduct of monetary policy in an open economy. Although the ECB has already stated that it will not follow any specific policy with regard to the exchange rate, it will inevitably monitor developments in this variable and take these into account when formulating policy. In order to keep the exchange rate fixed, the European monetary authority must set the money supply according to the following reaction function:

$$E = \overline{E}_0 \Rightarrow M_{eu} = \frac{\overline{M}_{eu0}}{\overline{M}_{us0}} M_{us}$$

$$\overline{E} = E \Rightarrow \overline{M}_{eu} = \frac{M_{eu}}{M_{us}} \overline{M}_{us}$$

Assuming that the US monetary authority changes its money stock permanently, that is, assuming that $\overline{\mu}^*$ is equal to one, then setting $\overline{\mu}$ equal to one is enough to ensure that the policy rule is observed in the long run.[32] In the short run, the

[32] Recall that $\overline{\mu}^* = \frac{\overline{M}_{us}}{M_{us}}$ and $\overline{\mu} = \frac{\overline{M}_{eu}}{M_{eu}}$.

effect of this policy on consumption and output is exactly the same as under inflation targeting. Since output prices are fixed in the short run, movements in the consumer price index are due to fluctuations in the exchange rate. However, if the US money shock is permanent, under this policy regime, prices will have to finally adjust in the long-run, such that:

$$\frac{\overline{P}^{EU}}{P^{EU}} = \frac{M_{us}}{\overline{M}_{us0}}$$

As in the inflation targeting framework, under a fixed exchange rate regime, the foreign monetary disturbances produce a proportional effect on consumption and output, but this policy can only fully stabilize utility if the weights of consumption and output/leisure in the utility function are the same. Alternatively, a combination policy which includes the nominal exchange rate, and either the money stock growth or the nominal interest rate, can do better.[33] Both types of combinations have already been introduced in the literature. A combination between the interest rate and the exchange rate, usually referred to as the *Monetary Conditions Index* (MCI) and already used in practice by the Bank of Canada, for instance, has been recently analyzed by Freeman (1994). Combinatorial rules specifying trade-offs of monetary growth against exchange rate deviations from a pre-specified path, which will be denoted *International Money Index*, have been defended by Artis and Currie (1981) because of their ability to control for losses in competitiveness. They show evidence which suggests that the pursuit of monetary targets has frequently been associated with substantial and persistent real appreciation (German and Switzerland between 1978 and 1979 and the UK in 1977 and 1979-80), which can be avoided by including the exchange rate in the targeting rule.

Targeting a *Monetary Conditions Index* can be defined as targeting the relationship of the type:

$$\left(\frac{E_{eu}}{E_{eu0}}\right)^{\alpha_{MCI}} = \left(\frac{1+i_{eu}}{1+\delta}\right)^{1-\alpha_{MCI}}$$

which minimizes the loss function, and where α_{MCI} determines the weight of each variable in the combination. If $\alpha_{MCI} = 1$ the combination policy is a pure fixed

[33] The first application of the instrument problem developed by Poole (1970) to the choice of an exchange rate regime can be found in Boyer (1978). Boyer uses a simple IS-LM open economy model to compare, in terms of output variability, a flexible regime with intervention regimes, operating either through open market operations or through fiscal and commercial policy instruments. As in Poole the optimal policies depend on the sources of shocks and the optimal intervention is a compromise between full flexibility and complete fixity. Parkin (1978) reviews the results assuming rational expectations and investigates whether the stabilization problem still holds. He finds that even if expectations are rational, the choice of the instrument will still determine which variable in the economy will cushion the shock, which is exactly the point that this paper, more explicitly, tries to make.

exchange rate policy, while if $\alpha_{MCI} = 0$ this reduces to a pure interest rate rule. In this case the value of α_{MCI} which insulates the economy from the foreign money supply shock is equal to:[34]

$$\alpha_{MCI} = \frac{(1-\gamma)(\rho-1)}{(1+(1-\gamma)(\rho-1))-\xi_{eu}}; \quad \xi_{eu} \neq \frac{1}{\gamma}$$

The *International Money Index* (IMI) targeting, on the other hand, can be defined as a relationship between the exchange rate and the money growth rate of the form:

$$\left(\frac{E}{E_0}\right)^{\alpha_{IMI}} = \left(\frac{M_{eu}}{M_{eu0}}\right)^{1-\alpha_{IMI}}$$

which minimizes the loss function, and where α_{IMI} determines the nature of the relationship between the two variables. The IMI policy collapses into a pure fixed exchange rate policy when $\alpha_{IMI} = 1$ and into a pure money supply rule when $\alpha_{IMI} = 0$. The value of α_{IMI} for which the loss function is minimized is now equal to:[35]

$$\alpha_{IMI} = \frac{(1-\gamma)\xi_{eu}}{(2-\gamma)\xi_{eu}-1}; \quad \xi_{eu} \neq \frac{1}{\gamma}$$

As in the case of the *two-pillar policy with the money growth rate* no knowledge of the parameter of risk aversion ρ is required to implement this policy rule. However a precise knowledge of γ (the world market share) is required, in this case.

Other possible rules could also be analyzed. Nominal income targeting, for instance can be obtained as special case of targeting an index containing the nominal exchange rate and the money stock. This can be seen by writing the expression for the short-run nominal income growth as follows:

$$\frac{P^{EU}}{\overline{P}_0^{EU}} \frac{Y_{eu}}{\overline{Y}_{eu0}} = \left(\frac{E}{\overline{E}_0}\right)^{1-\gamma} \frac{M_{eu}}{\overline{M}_{eu0}}$$

The intuition behind this is that, in the short run, the price index responds to changes in the nominal exchange rate while real output responds to changes in the money stock. Nominal income targeting however, implies maintaining a particular relationship between the exchange rate and the money stock which is exactly given by the relative share of US goods in the world market, as shown in equation (37).

$$\frac{P^{EU}}{\overline{P}_0^{EU}} \frac{Y_{eu}}{\overline{Y}_{eu0}} = 1 \Rightarrow \left(\frac{E}{\overline{E}_0}\right)^{1-\frac{1}{\gamma}} = \left(\frac{M_{eu}}{\overline{M}_{eu0}}\right)^{\frac{1}{\gamma}} \tag{37}$$

[34]The calculations are shown in Appendix A.5.
[35]See Appendix A.5 for details.

This relationship will only correspond to the optimal IMI described in section 5, if the relative weight between consumption and output in the utility function is equal to 1/2, that is, the output term in the steady state utility function has to have twice as much weight as the consumption term for a nominal income strategy to be able to fully stabilize utility. Recall that the optimal relationship between the exchange rate and the money shock, determining the IMI implies:

$$\alpha_{IMI} = \frac{(1-\gamma)\xi_{eu}}{(2-\gamma)\xi_{eu}-1}$$

Hence, targeting nominal income will only be equivalent to using the optimal IMI targeting provided $\alpha_{IMI} = 1-\frac{1}{\gamma}$, that is, as long as the following condition is satisfied:

$$\frac{(1-\gamma)\xi_{eu}}{(2-\gamma)\xi_{eu}-1} = 1-\frac{1}{\gamma} \Rightarrow \xi_{eu} = \frac{1}{2}$$

Although the discussion presented here has only focused on one type of disturbance, namely US money shocks, these are in fact quite interesting for analyzing monetary policy rules. These shocks are an example of a foreign disturbances in response to which the policy maker has an interest to focus on variables that reflect external economic conditions (consumer price indexes, the interest rate and the exchange rate). In addition because they are nominal shocks, they affect real variables only in the short run, which corresponds to the time horizon for which monetary policy is effective. Nevertheless, the next subsection outlines the results for other types of shocks.

5.2 Other shocks

The analysis illustrates in the context of an open economy, in which "home" consumption may differ from production, the existence of a trade-off in terms of welfare stabilization through the use of different policy rules. This trade off can be better resolved with "combination" rules.

On the contrary, if temporary US fiscal shocks are taken into account, the best policy alternative is always to let the money stock remain constant since, in the short run, neither European consumption nor European output respond to the US fiscal shocks while, in the long run, monetary policy is neutral. If, on the other hand, permanent European fiscal shocks are considered, then there is an additional trade-off between long-run and short-run movements in the real variables.[36] Fixing the money stock produces a larger overshooting/undershooting of output, in the case of "home" fiscal shocks. However, under interest rate targeting there is a short-run change in consumption, that only occurs in the long-run under a money growth rule, that is, when the nominal interest rate is kept fixed changes in consumption are anticipated one period relative to the case where the money stock is kept fixed. Appendix A.6 summarizes these results. In what concerns shocks to the parameters of the model, only shocks to the coefficient of risk aversion (ρ) or the market share (γ) could matter in answering the questions raised in the context of this model, because the nominal interest rate does not react to changes in the parameters of monopolistic competition in the labor market; but no parameter changes were analyzed here.

6 Conclusions

The purpose of this paper is to evaluate whether a monetary policy strategy consisting of multiple intermediate targets (such as money growth and inflation) can be superior to an adoption of a single target, such as inflation targeting. Three different policies – money growth targeting, interest rate targeting and inflation targeting – were compared in terms of their effectiveness in mitigating a foreign-induced (i.e., US) monetary shock. These alternative policies were ranked according to their ability to stabilize inflation and, alternatively, the utility of consumers. It was shown that, when the utility of consumers depends on both consumption and leisure, rather than targeting a single intermediate target, the central bank can achieve a better outcome, in terms of welfare stabilization, by combining two different intermediate targets. In terms of inflation stabilization, the paper confirmed that inflation targeting does achieve the best outcome, while combining inflation with the interest rate, or money growth, or combining money growth with the interest rate all yield somewhat higher inflation variability. Nonetheless, any of these combination policies, while sacrificing some price stability, may in fact achieve a better outcome in terms of full welfare stabilization. In order to achieve this outcome, policymakers need to know the underlying parameters of the model (e.g., the degree of risk aversion, the market power of workers, the intertemporal discount rate, etc.).

However, when uncertainty regarding these parameters is high, a poorly designed "combination" policy can exacerbate the effect of shocks on consumption and output. Interestingly, a policy combining the inflation rate and the money growth rate does

[36] Temporary home fiscal shocks are uninteresting because the nominal interest rate does not respond to temporary fiscal shocks, and fixing the nominal interest rate is equivalent to fixing the money stock.

not require the knowledge of the parameter of risk aversion to guarantee that welfare is stabilized successfully. Hence, in the context of uncertainty about the model's parameters, this combination policy would be preferred on the basis of its simplicity. This analysis, therefore, contributes toward understanding better some of the trade-offs faced by policymakers in open economies. In addition, the results provide some support for the current design of the ECB's operational framework which could be interpreted as focusing on two intermediate objectives, money growth and inflation, through the two-pillar strategy. This is because stabilizing money-growth, in addition to inflation, gives an additional degree of freedom to stabilize output. Although price stability is likely to remain the primary objective of the ECB, as mentioned before, monetary policy must "without prejudice of price stability (...) support the general economic policies in the Community..." (Article 2). Hence monitoring money, under certain assumptions about the shocks hitting the economy, may deliver a better outcome in terms of output stabilization which should allow the ECB to fulfill its secondary but nonetheless important commitment. Achieving more than one goal almost always requires more than one intermediate target.

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A Appendix

A.1 Solving the problem of consumers

The wealth $\Omega_{\tau-1}$, consisting of last period's wage income, lump-sum transfers, cash balances, and assets plus interest earnings that have not been spent on last period's consumption, can be formally written as:

$$\begin{aligned}\Omega(j)_{\tau-1} \equiv\ & \left(W_{eu}(j)_{\tau-1}L_{eu}(j)_{\tau-1} + P^{EU}_{\tau-1}T_{eu\tau-1}\right) + \left(M^{EU}_{eu}(j)_{\tau-1} + E_{\tau-1}M^{EU}_{us}(j)_{\tau-1}\right) + \\ & + \left(P^{EU}_{\tau}(1+r_{eu\tau-1})A^{EU}_{eu}(j)_{\tau-1} + P^{EU}_{\tau}(1+r_{us\tau-1})A^{EU}_{us}(j)_{\tau-1}\right) - \\ & - \left(P_{eu\tau-1}C^{EU}_{eu}(j)_{\tau-1} + E_{\tau-1}P_{us\tau-1}C^{EU}_{us}(j)_{\tau-1}\right)\end{aligned}$$

This section shows the conditions needed for deriving the solution of the model, for the case of Europe. The solution for the case of the US is analogous. Given that in the optimum the cash-in-advance constraints (4) and the budget constraint (5), will be binding, the Lagrangian for this problem can be simplified to the expression below:

$$\begin{aligned}\pounds_t =\ & \sum_{\tau=0}^{\infty}\beta^{\tau}\left[\frac{C^{EU}(j)^{1-\rho}_{\tau}}{1-\rho} + V(G_{eu\tau}) - \frac{k}{2}L_{eu}(j)^2_{\tau} + \lambda_{\tau}(\Lambda_{eut})\right] \\ \Lambda_{eut} \equiv\ & (L_{eu}(j)^{1-\frac{1}{\phi}}_{\tau-1}P_{eu\tau-1}Y_{\tau-1}^{\frac{1}{\phi}} + P^{EU}_{\tau-1}T_{eu\tau-1}) + \\ & + \left(P^{EU}_{\tau}(1+r_{eu\tau-1})A^{EU}_{eu}(j)_{\tau-1} + P^{EU}_{\tau}(1+r_{us\tau-1})A^{EU}_{us}(j)_{\tau-1}\right) - \\ & - \left(P_{eu\tau}C^{EU}_{eu}(j)_{\tau} + E_{\tau}P_{us\tau}C^{EU}_{us}(j)_{\tau} + P^{EU}_{\tau}A_{eu}(j)_{\tau} + P^{EU}_{\tau}A_{us}(j)_{\tau}\right)\end{aligned}$$

which has the following 'First Order Conditions' (FOCs):

$$C^{EU}(j)^{1-\rho}_{t}\frac{\gamma}{P_{eut}C^{EU}_{eu}(j)_t} = \lambda_t \tag{38}$$

$$C^{EU}(j)^{1-\rho}_{t}\frac{1-\gamma}{E_tP_{ust}C^{EU}_{us}(j)_t} = \lambda_t \tag{39}$$

$$-\lambda_t + \beta\lambda_{t+1}(1+r_{eut}) = 0 \tag{40}$$

$$-\lambda_t + \beta\lambda_{t+1}(1+r_{ust}) = 0 \tag{41}$$

$$-kL_{eu}(j)_t + \beta\lambda_{t+1}\frac{\phi-1}{\phi}L_{eu}(j)_t^{-\frac{1}{\phi}}P_{eut}Y_{eut}^{\frac{1}{\phi}} = 0 \tag{42}$$

Using equations (38) and (39) it is then possible to write that:

$$\frac{\gamma}{P_{eut}C^{EU}_{eu}(j)_t} = \frac{1-\gamma}{E_tP_{ust}C^{US}_{us}(j)_t}$$

Rearranging this relation yields the following relationships:

$$P_t^{EU}C_t^{EU} = P_{eut}C_{eu}^{EU}(j)_t + E_tP_{ust}C_{us}^{EU}(j)_t = \frac{1}{\gamma}P_{eut}C_{eu}^{EU}(j)_t = \frac{1}{1-\gamma}E_tP_{ust}C_{us}^{EU}(j)_t \tag{43}$$

which can be used together with equation(38) to obtain:

$$\lambda_t = C^{EU}(j)_t^{-\rho}$$

Arbitrage in the asset market determines that $r_{eut} = r_{ust} = r_t$, for all t. This follows from satisfying (40) and (41) simultaneously. Substituting $\lambda_t = C^{EU}(j)_t^{-\rho}$, for all t, into one of these last equations and into (42) yields, respectively, the usual Euler equation and a labor-leisure trade-off:

$$\left(\frac{C^{EU}(j)_{t+1}}{C^{EU}(j)_t}\right)^{\rho} = \beta(1+r_t)$$

$$L_{eu}(j)_t = \beta\frac{\phi-1}{k\phi}L_{eu}(j)_t^{-\frac{1}{\phi}}Y_{eut}^{\frac{1}{\phi}}\frac{P_{eut}}{P_t^{EU}}\frac{P_t^{EU}}{P_{t+1}^{EU}}C^{EU}(j)_{t+1}^{-\rho} = 0$$

Assuming symmetry the index j can drop out and $L_{eu}(j)_t = L_{eut} = Y_{eut}$. In this case the conditions for optimality become:

$$\left(\frac{C_{t+1}^{EU}}{C_t^{EU}}\right)^{\rho} = \beta(1+r_t) \tag{44}$$

$$Y_{eut} = \beta\Phi\frac{P_{eut}}{P_t^{EU}}\frac{P_t^{EU}}{P_{t+1}^{EU}}\left(C_{t+1}^{EU}\right)^{-\rho} \tag{45}$$

where $\Phi = \dfrac{\phi-1}{k\phi}$. Finally the transversality condition for this problem, which ensures that no wealth is accumulated at time $t+\tau$, when τ tends to infinity, is given below.

$$\lim_{\tau\to\infty}\frac{1}{\prod_{s=t+1}^{t+\tau}(1+r_s)}\left(A_{eut+\tau}^{EU} + A_{ust+\tau}^{EU} + \frac{M_{eut+\tau}^{EU} + E_{t+\tau}M_{ust+\tau}^{EU}}{P_{t+\tau}^{EU}}\right) = 0$$

A problem, in all respects analogous to this, is solved by US consumers.

A.2 Model Solutions

Let us first characterize the transition period. In this case, the Euler equation provides a link between the short run, when disturbances occur and the long run when adjustments have already taken place. From these equations and because arbitrage implies that the interest rate in real terms is equalized across the Atlantic, it follows that the following relation between European and US consumption changes must hold.

$$\frac{\overline{C}^{EU}}{C^{EU}} = \frac{\overline{C}^{US}}{C^{US}} \tag{46}$$

where, as indicated earlier, variables with a bar on top describe the long run and variables without a bar describe the short run. This condition, rearranged, shows that the international consumption ratios are the same in the long and in the short run. In addition, another link between the short run and the long run can be found in the current accounts for the first period of the new steady state:

$$\overline{NFA}^{EU} = (1 + r_{eu})NFA^{EU} + \frac{\overline{P}_{eu}(\overline{Y}_{eu} - \overline{G}_{eu})}{\overline{P}^{EU}} - \overline{C}^{EU} \tag{47}$$

$$\overline{NFA}^{US} = -(1 + r_{us})NFA^{US} + \frac{P_{us}(Y_{us} - G_{us})}{P^{US}} - \overline{C}^{US} \tag{48}$$

where, assuming that the initial stock of external debt is zero, that is, $NFA_0^{EU} = NFA_0^{EU} = 0$, the consumption indexes in the short run must satisfy:

$$C^{EU} = \frac{P_{eu}(Y_{eu} - G_{eu})}{P^{EU}} - NFA^{EU} \qquad C^{US} = \frac{P_{us}(Y_{us} - G_{us})}{P^{US}} - NFA^{US} \tag{49}$$

$$\overline{C}^{EU} = \delta\overline{NFA}^{EU} + \frac{\overline{P}_{eu}(\overline{Y}_{eu} - \overline{G}_{eu})}{\overline{P}^{EU}} \qquad \overline{C}^{US} = \delta\overline{NFA}^{US} + \frac{\overline{P}_{us}(\overline{Y}_{us} - \overline{G}_{us})}{\overline{P}^{US}} \tag{50}$$

Substituting $\overline{C}^{EU}$ derived from equation (50) into equation (47) it can be shown that $(1 + \delta)\overline{NFA}^{EU} = (1 + r_{eu})NFA^{EU}$. Substituting $P_{eu}(Y_{eu} - G_{eu})$, $P_{us}(Y_{us} - G_{us})$, $\overline{P}_{eu}(\overline{Y}_{eu} - \overline{G}_{eu})$ and $\overline{P}_{us}(\overline{Y}_{us} - \overline{G}_{us})$ using the market clearing conditions (8), into equations (49) and (50) and dividing the European by the US current account, yields the following equilibrium relations:

$$\frac{C^{EU} + NFA^{EU}}{C^{US} + NFA^{US}} = \frac{\gamma}{1-\gamma} \qquad \frac{\overline{C}^{EU} - \delta\overline{NFA}^{EU}}{\overline{C}^{US} - \delta\overline{NFA}^{US}} = \frac{\gamma}{1-\gamma} \tag{51}$$

Recall that $NFA_t^{US} = -NFA_t^{EU}$, for all t. Given that $(1 + \delta)\overline{NFA}^{EU} = (1 + r_{eu})NFA^{EU}$, equations (51), together with equations (46) imply that $\overline{NFA}^{EU} = NFA^{EU} = 0$ and $\overline{NFA}^{US} = NFA^{US} = 0$. Substituting these results into equations (51) gives the ratios of European to US consumption, in the long run and in the short run, which are always constant and equal to $\gamma/(1-\gamma)$.

$$\frac{C^{EU}}{C^{US}} = \frac{\overline{C}^{EU}}{\overline{C}^{US}} = \frac{\gamma}{1-\gamma} \tag{52}$$

In the case of the model presented here, relaxing the assumption that the initial external debt is zero could allow temporary shocks, with an impact in the real interest rate, to induce current account changes, but for simplicity this alternative was not pursued.[37]

To help solve the model, it is simpler to define the following indexes of fiscal stance: $g_{eu} \equiv Y_{eu}/(Y_{eu} - G_{eu})$ and $g_{us} \equiv Y_{us}/(Y_{us} - G_{us})$. Notice that g_{eu} (g_{us}) is equal to one when G_{eu} (G_{us}) is zero and is increasing in G_{eu}/Y_{eu} (G_{us}/Y_{us}). In addition define also the following world aggregates (x_{wt}) and the following relative indexes (x_{Rt}):

$$x_{wt} \equiv (x_{eut})^{\gamma}(x_{ust})^{1-\gamma} \qquad\qquad x_{Rt} \equiv x_{eut}/x_{ust} \tag{53}$$

where x_{eu} can be any of Europe's variables. Given a variable x_{eu}, x_{us} stands for its US analogue. Notice that $x_{eut} = (x_{Rt})^{1-\gamma}x_{wt}$ and $x_{ust} = (x_{Rt})^{-\gamma}x_{wt}$. In what concerns the parameters of the model, for any parameters α (referring to the European economy) and α^* (referring to the US economy), similar aggregates will be defined whenever it is convenient, such that: $\alpha_w \equiv (\alpha)^{\gamma}(\alpha^*)^{1-\gamma}$ and $\alpha_{Rt} \equiv \alpha/\alpha^*$.

Now it is possible to solve for the long-run equilibrium in this model. First recall the goods market clearing conditions (8). In the long run these conditions become:

$$\frac{\overline{P}_{eu}(\overline{Y}_{eu} - \overline{G}_{eu})}{\overline{P}^{EU}} = \gamma(\overline{C}^{EU} + \overline{C}^{US})$$

$$\frac{\overline{P}_{us}(\overline{Y}_{us} - \overline{G}_{us})}{\overline{P}^{US}} = (1-\gamma)(\overline{C}^{EU} + \overline{C}^{US})$$

Substituting $\overline{C}^{US} = \dfrac{1-\gamma}{\gamma}\overline{C}^{EU}$ in the European goods market equilibrium and $\overline{C}^{EU} = \dfrac{\gamma}{1-\gamma}\overline{C}^{US}$ in the US goods market equilibrium and substituting $(\overline{Y}_{eu} - \overline{G}_{eu})$ by $\overline{Y}_{eu}/\overline{g}_{eu}$ and $(\overline{Y}_{us} - \overline{G}_{us})$ by $\overline{Y}_{us}/\overline{g}_{us}$, according to the notation introduced before, yields:

$$\frac{\overline{P}_{eu}\overline{Y}_{eu}}{\overline{P}^{EU}\overline{g}_{eu}} = \overline{C}^{EU} \qquad\qquad \frac{\overline{P}_{us}\overline{Y}_{us}}{\overline{P}^{US}\overline{g}_{us}} = \overline{C}^{US} \tag{54}$$

Replacing these results in the long-run versions of the quality-theory-of-money equations (10), gives the equilibrium locus MC, providing a relationship between the real stock of money and consumption:

$$MC: \quad \frac{\overline{M}_{eu}}{\overline{P}^{EU}} = \frac{\overline{P}_{eu}}{\overline{P}^{EU}}\frac{\overline{Y}_{eu}}{\overline{g}_{eu}} \Rightarrow \frac{\overline{M}_{eu}}{\overline{P}^{EU}} = \overline{C}^{EU} \tag{55}$$

[37] See Obstfeld and Rogoff (1996) for evidence that this is the case.

and analogously for the US:

$$\frac{\overline{M}_{us}}{\overline{P}^{US}} = \frac{\overline{P}_{us}}{\overline{P}^{US}}\frac{\overline{Y}_{us}}{\overline{g}_{us}} \Rightarrow \frac{\overline{M}_{us}}{\overline{P}^{US}} = \overline{C}^{US} \tag{56}$$

Notice that, using (52) and dividing the two previous relations gives a solution for the long-run exchange rate in terms of the relative money supplies, because PPP holds for the aggregate price indexes:

$$\frac{\overline{M}^{EU}/\overline{P}^{EU}}{\overline{M}^{US}/\overline{P}^{US}} = \frac{\overline{C}^{EU}}{\overline{C}^{US}} \Rightarrow \overline{E} = \frac{1-\gamma}{\gamma}\overline{M}_R \tag{57}$$

In the long run, both the labor-leisure trade off and the goods market equilibrium will be binding in the two economies. These two relations determine two other equilibrium loci relationships between consumption and output, YC_1 and YC_2, which are enough to determine the long-run levels for these variables in Europe and in the US. In the long run, money will have no effect on consumption and output. It will not have an effect on the terms of trade either. This can be observed by dividing both the goods market equilibrium equations derived in (54) and by using PPP again, in order to obtain equation (58):

$$\frac{\overline{P}_{eu}}{\overline{EP}_{us}} = \frac{\overline{g}_R}{\overline{Y}_R}\frac{\gamma}{1-\gamma} \tag{58}$$

The first of the the loci relating consumption and output is derived from the goods market equilibrium conditions alone. To find this equilibrium schedule, notice that, by definition of the price index is equal to:

$$\frac{\overline{P}^{EU}}{\overline{P}_{eu}} = \frac{1}{\gamma_w}\left(\frac{\overline{EP}_{us}}{\overline{P}_{eu}}\right)^{1-\gamma} \tag{59}$$

Substituting the terms of trade by the expression given in equation (58) and substituting the resulting expression for $\overline{P}^{EU}/\overline{P}_{eu}$ into the European goods market equilibrium given by equation (54), yields:

$$\overline{Y}_{eu} = \frac{1}{\gamma_w}\left(\frac{\overline{Y}_R}{\overline{g}_R}\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\overline{g}_{eu}\overline{C}^{EU} \tag{60}$$

The ratio $\overline{g}_R/\overline{Y}_R$ can now be calculated by dividing the long-run European labor-leisure trade-off by its US analogue, which can be obtained from (45), letting $P_t^j/P_{t+1}^j = 1$, $j = EU, US$ and substituting all variables z_t by their long-run values $\overline{z}$.

$$\begin{aligned}
\overline{Y}_{eu} &= \beta\Phi\frac{\overline{P}_{eu}}{\overline{P}^{EU}}\left(\overline{C}^{EU}\right)^{-\rho} \\
\overline{Y}_{us} &= \beta\Phi^*\frac{\overline{P}_{us}}{\overline{P}^{US}}\left(\overline{C}^{US}\right)^{-\rho}
\end{aligned}$$

Notice that substituting $\left(\overline{P}_{eu}/\overline{P}^{EU}\right)$ and $\left(\overline{P}_{us}/\overline{P}^{US}\right)$, using equations (54), yields:

$$\overline{Y}_{eu} = \beta\Phi\left(\overline{C}^{EU}\right)^{1-\rho}\frac{\overline{g}_{eu}}{\overline{Y}_{eu}}$$

$$\overline{Y}_{us} = \beta\Phi^{*}\left(\overline{C}^{US}\right)^{1-\rho}\frac{\overline{g}_{us}}{Y_{us}}$$

Dividing now both equations, substituting C_R by $\dfrac{\gamma}{1-\gamma}$ and rearranging them gives $\overline{g}_R/\overline{Y}_R$:

$$\frac{\overline{Y}_R}{\overline{g}_R} = (\Phi_R)^{\frac{1}{2}}\left(\frac{\gamma}{1-\gamma}\right)^{\frac{1-\rho}{2}}(\overline{g}_R)^{-\frac{1}{2}} \tag{61}$$

Introducing this result in equation (60) gives the locus YC_1:

$$YC_1: \quad \overline{Y}_{eu} = \frac{(\Phi_R)^{\frac{1-\gamma}{2}}}{\gamma_w}\left(\frac{\gamma}{1-\gamma}\right)^{-\frac{(1-\gamma)(1+\rho)}{2}}(\overline{g}_{us})^{\frac{1-\gamma}{2}}(\overline{g}_{eu})^{\frac{1+\gamma}{2}}\overline{C}^{EU} \tag{62}$$

According to this equation, when European agents increase their consumption, the demand for European goods increases proportionally. Additionally, the effects of fiscal expansions in Europe and in the US differ. A fiscal expansion in the US improves Europe's terms of trade, shifting demand towards European goods, while a fiscal expansion at home has two opposite effects: it directly increases demand for European goods, on the one hand, but on the other hand it worsens Europe's terms of trade and, therefore shifts some demand towards US goods. Here the first effect is dominant.

The second schedule can be obtained by calculating world output from the labor leisure trade-offs:

$$\begin{aligned}
\overline{Y}_w &= \left(\beta\Phi\frac{\overline{P}_{eu}}{\overline{P}^{EU}}\left(\overline{C}^{EU}\right)^{-\rho}\right)^{\gamma}\left(\beta\Phi^{*}\frac{\overline{P}_{us}}{\overline{P}^{US}}\left(\overline{C}^{US}\right)^{-\rho}\right)^{1-\gamma} \qquad (63)\\
&= \left(\beta\Phi\gamma_w\left(\frac{\overline{P}_{eu}}{\overline{EP}_{us}}\right)^{1-\gamma}\left(\overline{C}^{EU}\right)^{-\rho}\right)^{\gamma}\left(\beta\Phi^{*}\gamma_w\left(\frac{\overline{EP}_{us}}{\overline{P}_{eu}}\right)^{\gamma}\left(\overline{C}^{EU}\right)^{-\rho}\right)^{1-\gamma}\\
&\Rightarrow \; \overline{Y}_w = \beta\Phi_w\gamma_w\left(\overline{C}_w\right)^{-\rho}
\end{aligned}$$

where $\overline{C}_w = \left(\overline{C}^{EU}\right)^{\gamma}\left(\overline{C}^{US}\right)^{1-\gamma}$. Replacing $\overline{C}^{EU}$ and $\overline{C}^{US}$ using equations (54) yields:

$$\begin{aligned}
\overline{C}_w &= \left(\frac{\overline{P}_{eu}\overline{Y}_{eu}}{\overline{P}^{EU}\overline{g}_{eu}}\right)^{\gamma}\left(\frac{\overline{P}_{us}\overline{Y}_{us}}{\overline{P}^{US}\overline{g}_{us}}\right)^{1-\gamma} \\
&= \left(\frac{\overline{P}_{us}(\overline{P}^{EU}/\overline{P}^{US})}{\overline{P}_{eu}}\frac{\overline{g}_{eu}/\overline{g}_{us}}{\overline{Y}_{eu}/\overline{Y}_{us}}\right)^{1-\gamma}\overline{C}^{EU} \\
&= \left(\frac{\overline{EP}_{eu}}{\overline{P}_{us}}\frac{\overline{g}_R}{\overline{Y}_R}\right)^{1-\gamma}\overline{C}^{EU}
\end{aligned}$$

Using now equation (58) to substitute for the term in brackets, allows to write:

$$\overline{C}_w = \left(\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\overline{C}^{EU} \tag{64}$$

It is also possible to calculate $\overline{Y}_{eu}$ using the fact that $\overline{Y}_{eu} = (\overline{Y}_R)^{1-\gamma}\overline{Y}_w$, by substituting the solutions for $\overline{Y}_R$ and $\overline{Y}_w$ from equations (61) and (63) respectively:

$$\overline{Y}_{eu} = \left((\Phi_R)^{1/2}\left(\frac{\gamma}{1-\gamma}\right)^{\frac{1+\rho}{2}}(\overline{g}_R)^{\frac{1}{2}}\right)^{1-\gamma}\beta\Phi_w\gamma_w(\overline{C}_w)^{-\rho}$$

Substituting $\overline{C}_w$ from equation (64) and rearranging yields the locus YC_2:

$$YC_2: \quad \overline{Y}_{eu} = \gamma_w(\Phi_R)^{\frac{1-\gamma}{2}}\beta\Phi_w\left(\frac{\gamma}{1-\gamma}\right)^{\frac{(1+\rho)(1-\gamma)}{2}}(\overline{g}_R)^{\frac{1-\gamma}{2}}(\overline{C}^{EU})^{-\rho} \tag{65}$$

Here an increase in long-run consumption decreases the marginal benefit from consuming, such that agents prefer to work less, reducing output. On the other hand, a fiscal expansion (contraction) in Europe (in the US) results in an appreciation of the home currency, accompanied by an increase in real wages which creates incentives for consumers to supply more labor. Equations (62) and (65) form a system of two equations which can be solved for $\overline{Y}$ and $\overline{C}$ in terms of the parameters and the monetary and fiscal stances:

$$\begin{aligned}
(62) \quad = \quad (65) \Rightarrow \left(\overline{C}^{EU}\right)^{1+\rho} &= \left(\frac{\gamma}{1-\gamma}\right)^{(1+\rho)(1-\gamma)}(\gamma_w)^2\beta\Phi_w(\overline{g}_w)^{-1} \\
\Rightarrow \overline{C}^{EU} &= \left(\frac{\gamma}{1-\gamma}\right)^{(1-\gamma)}(\gamma_w)^{\frac{2}{1+\rho}}(\beta\Phi_w)^{\frac{1}{1+\rho}}(\overline{g}_w)^{-\frac{1}{1+\rho}}
\end{aligned}$$

Defining $\Gamma = (\gamma/(1-\gamma))^{(1-\gamma)}\gamma_w^{\frac{2}{1+\rho}}$ and $a_c = \Gamma(\Phi_w)^{\frac{1}{1+\rho}}$ gives equation (11) in the text. Substituting now $\overline{C}$ in equation (62):

$$\begin{aligned}
\overline{Y}_{eu} &= \frac{(\Phi_R)^{\frac{1-\gamma}{2}}}{\gamma_w}\left(\frac{\gamma}{1-\gamma}\right)^{-\frac{(1-\gamma)(1+\rho)}{2}}(\overline{g}_{us})^{\frac{1-\gamma}{2}}(\overline{g}_{eu})^{\frac{1+\gamma}{2}}\left(\frac{\gamma}{1-\gamma}\right)^{(1-\gamma)}(\gamma_w)^{\frac{2}{1+\rho}}(\beta\Phi_w)^{\frac{1}{1+\rho}}(\overline{g}_w)^{-\frac{1}{1+\rho}} \\
\Rightarrow \ \overline{Y}_{eu} &= \left(\frac{\gamma}{1-\gamma}\right)^{-\frac{(1-\gamma)(1+\rho-2)}{2}}(\gamma_w)^{\left(\frac{2}{1+\rho}-1\right)}\Phi^{\frac{1}{2}}(\Phi_w)^{\left(\frac{1}{1+\rho}-\frac{1}{2}\right)}\beta^{\frac{1}{1+\rho}}(\overline{g}_{eu})^{\frac{1}{2}}(\overline{g}_w)^{\left(-\frac{1}{2}-\frac{1}{1+\rho}\right)} \\
\Rightarrow \ \overline{Y}_{eu} &= \left(\frac{\gamma}{1-\gamma}\right)^{\frac{(1-\gamma)(1-\rho)}{2}}(\gamma_w)^{\frac{1-\rho}{1+\rho}}\Phi^{\frac{1}{2}}(\Phi_w)^{\frac{1-\rho}{2(1+\rho)}}\beta^{\frac{1}{1+\rho}}(\overline{g}_{eu})^{\frac{1}{2}}(\overline{g}_w)^{-\frac{1-\rho}{2(1+\rho)}}
\end{aligned}$$

Simplifying according to the notation introduced before yields:

$$\overline{Y}_{eu} = \Gamma^{\frac{1-\rho}{2}}\Phi^{\frac{1}{2}}(\Phi_w)^{\frac{1-\rho}{2(1+\rho)}}\beta^{\frac{1}{1+\rho}}(\overline{g}_{eu})^{\frac{1}{2}}(\overline{g}_w)^{-\frac{1-\rho}{2(1+\rho)}}$$

Defining now the parameter $a_y \equiv \Gamma^{\frac{1-\rho}{2}}\Phi^{\frac{1}{2}}(\Phi_w)^{\frac{1-\rho}{2(1+\rho)}}$ allows one to obtain equation (12) in the main text. From the money demand equation given by the cash-in-advance constraint we can also write:

$$\frac{\overline{M}_{eu}}{\overline{P}^{EU}} = a_c\beta^{\frac{1}{1+\rho}}(\overline{g}_w)^{-\frac{1}{1+\rho}}$$

Rearranging this expression gives the same solution for the long-run European price index as given by equation (13). Using equation (58) in the main text and equation (61) it is now possible to derive the solution for the terms of trade:

$$\begin{aligned}
\frac{\overline{EP}_{us}}{\overline{P}_{eu}} &= \frac{1-\gamma}{\gamma}(\Phi_R)^{\frac{1}{2}}\left(\frac{\gamma}{1-\gamma}\right)^{\frac{1-\rho}{2}}(\overline{g}_R)^{-\frac{1}{2}} \\
&\Rightarrow \ \frac{\overline{EP}_{us}}{\overline{P}_{eu}} = (\Phi_R)^{\frac{1}{2}}\left(\frac{\gamma}{1-\gamma}\right)^{-\frac{1+\rho}{2}}(\overline{g}_R)^{-\frac{1}{2}}
\end{aligned}$$

Defining also $a_p \equiv (\Phi_R)^{\frac{1}{2}}(\gamma/(1-\gamma))^{-\frac{1+\rho}{2}}$ gives the solution (16) in the main text. The expression for the long-run price of the home good can be found through the relation (59):

$$\begin{aligned}
\overline{P}_{eu} &= \gamma_w\left((\Phi_R)^{-\frac{1}{2}}\left(\frac{\gamma}{1-\gamma}\right)^{\frac{1+\rho}{2}}(\overline{g}_R)^{\frac{1}{2}}\right)^{1-\gamma}\overline{P}^{EU} \\
\overline{P}_{eu} &= \gamma_w(\Phi_R)^{-\frac{1-\gamma}{2}}\left(\frac{\gamma}{1-\gamma}\right)^{\frac{(1+\rho)(1-\gamma)}{2}}(\overline{g}_R)^{\frac{1-\gamma}{2}}(a_c)^{-1}\beta^{-\frac{1}{1+\rho}}(\overline{g}_w)^{\frac{1}{1+\rho}}\overline{M}_{eu} \\
\overline{P}_H &= (a_c)^{\rho}a_y\Phi^{-1}(a_c)^{-1}\beta^{-\frac{1}{1+\rho}}(\overline{g})^{\frac{1}{2}}(\overline{g}_w)^{\frac{1-\rho}{2(1+\rho)}}\overline{M}_{eu}
\end{aligned}$$

In the short-run, however, during the period when prices remain fixed, consumption levels are simply determined by the cash-in-advance constraints, and therefore by the short-run analogue of the locus (55), such that:

$$C^{EU} = \frac{M_{eu}}{P^{EU}} \tag{66}$$

where P^{EU} is determined by the trading prices of goods and by the relative short-run money supplies, since the result $E = \frac{1-\gamma}{\gamma} M_R$, also holds in the short run.[38] Because the prices of goods do not adjust, during this period, they remain equal to their initial levels which correspond to the initial steady state solution. Therefore, $P_{eu} = \overline{P}_{eu0}$ and $P_{us} = \overline{P}_{us0}$, where $\overline{P}_{eu0}$ and $\overline{P}_{us0}$ are given by the same expressions as $\overline{P}_{eu}$ and $\overline{P}_{us}$, with the fiscal and monetary stances at the initial levels $\overline{g}_{eu0}$, $\overline{g}_{us0}$, $\overline{M}_{eu0}$, $\overline{M}_{us0}$ and with the exchange rate given by $\overline{E}_0 = \frac{1-\gamma}{\gamma}\overline{M}_{R0}$, such that:

$$P_{eu} = (a_c)^{\rho-1} a_y \Phi^{-1} \beta^{-\frac{1}{1+\rho}} (\overline{g}_{eu0})^{\frac{1}{2}} (\overline{g}_{w0})^{\frac{1-\rho}{2(1+\rho)}} \overline{M}_{eu0} \tag{67}$$

$$P_{us} = a_p (\overline{g}_{R0})^{-\frac{1}{2}} P_{eu} \frac{\gamma}{1-\gamma} (\overline{M}_{R0})^{-1} \tag{68}$$

To obtain the European price index from these equations, notice that:

$$(P_{eu})^{\gamma} \left(\frac{1-\gamma}{\gamma} P_{us}\right)^{1-\gamma} = \beta^{-\frac{1}{1+\rho}} (a_c)^{\rho-1} a_y \Phi^{-1} (a_p)^{1-\gamma} (\overline{g}_{w0})^{\frac{1}{1+\rho}} \overline{M}_{w0} \tag{69}$$

Substituting this expression into the definition of the short-run price gives:

$$P^{EU} = \frac{1}{\gamma_w} \beta^{-\frac{1}{1+\rho}} (a_c)^{\rho-1} a_y \Phi^{-1} (a_p)^{1-\gamma} (\overline{g}_{w0})^{\frac{1}{1+\rho}} \overline{M}_{w0} (M_R)^{1-\gamma}$$

Notice that, since $a_y = (a_c)^{\frac{1-\rho}{2}} \Phi^{\frac{1}{2}}$ and $\gamma_w^{-1} (a_p)^{(1-\gamma)} = (a_c)^{-\frac{1+\rho}{2}} \Phi^{\frac{1}{2}}$, the previous expression can be simplified to yield the solution (19). Substituting that solution into (66) allows to write home consumption as a function of the world money supply as in equation (17). Output for the short run can be obtained from the home good market equilibrium. Recall that the home goods market equilibrium gives output as a function of (P^{EU}/P_{eu}), C^{EU} and g_{eu}.

$$\frac{P_{eu} Y_{eu}}{P^{EU} g_{eu}} = C^{EU} \Rightarrow Y_{eu} = \frac{P^{EU}}{P_{eu}} g_{eu} C^{EU} \tag{70}$$

[38] This result can be found again by dividing the cash-in-advance constraints of both countries in the short-run and by substituting $C^{EU} = \frac{\gamma}{1-\gamma} C^{US}$.

Dividing (19) by (67) allows to write (P^{EU}/P_{eu}) as follows:

$$
\begin{aligned}
\frac{P^{EU}}{P_{eu}} &= \frac{(a_c)^{-1}\beta^{-\frac{1}{1+\rho}}\left(\overline{g}_{w0}\right)^{\frac{1}{1+\rho}}\overline{M}_{w0}\left(M_R\right)^{1-\gamma}}{(a_c)^{\rho-1}a_y\Phi^{-1}\beta^{-\frac{1}{1+\rho}}\left(\overline{g}_{eu0}\right)^{\frac{1}{2}}\left(\overline{g}_{w0}\right)^{\frac{1-\rho}{2(1+\rho)}}\overline{M}_{eu0}} \\
&\Rightarrow \frac{P^{EU}}{P_{eu}} = (a_c)^{-\rho}(a_y)^{-1}\Phi^{1}\left(\overline{g}_{eu0}\right)^{-\frac{1}{2}}\left(\overline{g}_{w0}\right)^{\frac{1}{2}}\left(\frac{M_R}{\overline{M}_{R0}}\right)^{1-\gamma}
\end{aligned}
$$

Substituting this result into equation (70) and substituting C^{EU} by the result found in equation (17) yields:

$$
\begin{aligned}
Y_{eu} &= (a_c)^{1-\rho}(a_y)^{-1}\Phi\beta^{\frac{1}{1+\rho}}\left(\overline{g}_{eu0}\right)^{-\frac{1}{2}}\left(\overline{g}_{w0}\right)^{-\frac{1-\rho}{2(1+\rho)}} g_{eu}\frac{M_{eu}}{\overline{M}_{eu0}} \\
&\Rightarrow Y_{eu} = (a_c)^{1-\rho}(a_y)^{-1}\Phi\beta^{\frac{1}{1+\rho}}\left(\overline{g}_{eu0}\right)^{\frac{1}{2}}\left(\overline{g}_{w0}\right)^{-\frac{1-\rho}{2(1+\rho)}}\frac{M_{eu}}{\overline{M}_{eu0}}\frac{g_{eu}}{\overline{g}_{eu0}}
\end{aligned}
$$

Notice now that $(a_4)^{1-\rho}\Phi = (a_5)^2$, hence the previous expression can be simplified to yield the solution for the short-run level of European output, shown in equation (18) in the main text.Solutions in all respects analogous to these can be found for the case of the US, but will be omitted here for simplicity of exposition.

A.3 Inflation Variation

Under inflation targeting, price changes are zero, but under both money growth targeting and interest rate prices will change in response to shocks. Under money growth targeting, the aggregate price index changes in the short-run and this effect is offset by an opposite move in prices in the long-run. Under nominal interest rate targeting prices change only in the short run and this effect is not reversed. Overall the total price variability can be compared using useful log-linearizations:

$$
\begin{aligned}
&\frac{P^{EU}}{\overline{P}_0^{EU}} - 1 = \pi_{\overline{M}} \simeq \ln(1+\pi_{\overline{M}}) = -(1-\gamma)\ln\left(\frac{M_{us}}{\overline{M}_{us0}}\right) \\
&\frac{\overline{P}^{EU}}{P^{EU}} - 1 = \pi_{\overline{M}} \simeq \ln(1+\pi_{\overline{M}}) = (1-\gamma)\ln\left(\frac{M_{us}}{\overline{M}_{us0}}\right)
\end{aligned}
\tag{71}
$$

$$
\begin{aligned}
&\frac{P^{EU}}{\overline{P}_0^{EU}} - 1 = \pi_{\overline{i}} \simeq \ln(1+\pi_{\overline{i}}) = -\frac{1-\gamma}{\gamma}\ln\left(\frac{M_{us}}{\overline{M}_{us0}}\right) \\
&\frac{\overline{P}^{EU}}{P^{EU}} = \pi_{\overline{i}} \simeq \ln(1+\pi_{\overline{i}}) = 0
\end{aligned}
\tag{72}
$$

Using expressions (71) and (72) it is possible to derive the inflation variability under both regimes, such that:

$$
V[\ln(1+\pi_{\overline{M}})] = \left[-(1-\gamma)\ln\left(\frac{M_{us}}{\overline{M}_{us0}}\right)\right]^2 + \left[(1-\gamma)\ln\left(\frac{M_{us}}{\overline{M}_{us0}}\right)\right] = 2(1-\gamma)^2\left[\ln\left(\frac{M_{us}}{\overline{M}_{us0}}\right)\right]^2
$$

$$V[\ln(1+\pi_{\bar{i}})] = \left[-\frac{1-\gamma}{\gamma}\ln\left(\frac{M_{us}}{\overline{M}_{us0}}\right)\right]^2 = \left(\frac{1-\gamma}{\gamma}\right)^2\left[\ln\left(\frac{M_{us}}{\overline{M}_{us0}}\right)\right]^2$$

Comparing the two expression we can conclude that:

$$V[\ln(1+\pi_{\overline{M}})] < V[\ln(1+\pi_{\bar{i}})] \Longrightarrow \gamma < \frac{1}{\sqrt{2}}$$

A.4 The Welfare Analysis

The welfare analysis is based on minimizing deviations in the utility of European consumers. In order to determine an expression for this variation in terms of policy variables, define the steady-state level of the intertemporal utility function, excluding utility from public spending as $\widetilde{U}_0^{EU}$, such that:[39]

$$\widetilde{U}_0^{EU} = \frac{1}{1-\beta}\widetilde{u}_0^{EU}$$

where $\widetilde{u}_0^{EU} = (1-\rho)^{-1}\left(C_0^{EU}\right)^{1-\rho} - K/2\,(Y_{eu0})^2$. After the shock, since in the long-run consumption and output are not affected by monetary shocks and are equal to their initial levels, $\widetilde{U}$ becomes:

$$\widetilde{U}^{EU} = \widetilde{u}_0^{EU} + \beta\widetilde{u}^{EU} + \frac{\beta^2}{1-\beta}\widetilde{u}_0^{EU}$$

where $\widetilde{u}^{EU} = (1-\rho)^{-1}\left(C_0^{EU}\right)^{1-\rho} - K/2\,(Y_{eu})^2$. Notice that the difference between $\widetilde{U}^{EU}$ and $\widetilde{U}_0^{EU}$ is simply:

$$\widetilde{U}^{EU} - \widetilde{U}_0^{EU} = \beta\left(\widetilde{u}^{EU} - \widetilde{u}_0^{EU}\right) \tag{73}$$

Substituting now $\widetilde{u}_0^{EU}$ and $\widetilde{u}^{EU}$ gives:

$$U - U_0 = \beta\left(\frac{\left(C^{EU}\right)^{1-\rho} - \left(C_0^{EU}\right)^{1-\rho}}{1-\rho} - \frac{K}{2}\left((Y_{eu})^2 - (Y_{eu0})^2\right)\right)$$

Substituting now C^{EU} and Y_{eu} by their short-run solutions (17) and (18) respectively and rearranging yields:

$$\begin{aligned} U - U_0 &= \beta\left(\frac{C_0^{1-\rho}}{1-\rho}\left[\left(\frac{M_w}{M_{w0}}\right)^{1-\rho} - 1\right] - \frac{K}{2}Y_0^2\left[\left(\frac{M}{M_0}\right)^2\left(\frac{g}{g_0}\right)^2 - 1\right]\right) \\ &\simeq \beta\left(C_0^{1-\rho}\ln\left(\frac{M_w}{M_{w0}}\right) - KY_0^2\left[\ln\left(\frac{M}{M_0}\right) + \ln\left(\frac{g}{g_0}\right)\right]\right) \end{aligned}$$

using $x \simeq \ln(x+1)$. When $g = g_0$, the loss function $L_{eu} = E\left[(U-U_0)^2\right]$ is given by expression (32) in the text.

[39] The utility from public spending is additively separable and will be the same under the two policy rules, therefore it will not affect the ranking of the policy regimes.

A.5 Deriving the Optimal Combinations:

In the case of a "Poole" type of policy, combining the nominal interest rate and the money growth rate, the parameter a determines the optimal relationship between the variables. In order to obtain a solution for a which minimizes the loss function notice that, for any α, according to equation (23), the combination policy implies that:

$$\left(\frac{M_{eu}}{M_{eu0}}\right) = \left(\frac{M_{us}}{M_{us0}}\right)^{-\frac{(1-\gamma)(\rho-1)(1-\alpha)}{\alpha+\gamma(\rho-1)(1-\alpha)}} \qquad \frac{M_w}{M_{w0}} = \left(\frac{M_{us}}{M_{us0}}\right)^{\frac{\alpha(1-\gamma)}{\alpha+\gamma(\rho-1)(1-\alpha)}}$$

In addition, for any value of α, there is a corresponding loss function of the form:

$$L_{i/M}|_{eu} = (\beta(1-\gamma))^2 \left(\frac{\alpha(C_0^{EU})^{1-\rho} + (1-\alpha)(\rho-1)K\,(Y_{eu0})^2}{\alpha+\gamma(\rho-1)(1-\alpha)}\right)^2 \sigma^2_{M_{us}}$$

Notice that there is a value of the parameter α for which the loss function is zero for any value of $\sigma^2_{M_{us}}$, which is given by:

$$\begin{aligned} \alpha(C_0^{EU})^{1-\rho} + (1-\alpha)(\rho-1)K\,(Y_{eu0})^2 &= 0; \alpha+\gamma(\rho-1)(1-\alpha) \neq 0 \\ &\Rightarrow \alpha = \frac{(\rho-1)}{(\rho-1)-\xi_{eu}};\ \xi_{eu} \neq \frac{1}{\gamma} \end{aligned}$$

where ξ_{eu}, is defined as before in (34). If $\xi_{eu} = 1/\gamma$, that is, if $K\,(Y_{eu0})^2 = \gamma(C_0^{EU})^{1-\rho} = \varpi^*$, for any value α, including 0 and 1, the loss will be always equal to $\left(\frac{1-\gamma}{\gamma}\beta\varpi^*\right)^2 \sigma^2_{M_{us}}$.

Similarly, in the case of a "*two-pillar policy with the nominal interest rate*", for any value of the parameter $\alpha_{\pi i}$, which determines in this case the relationship to be maintained between inflation and the nominal interest rate, the loss function will be given by:[40]

$$L_{i/\pi}|_{eu} = \beta^2 \left(\frac{\alpha_{\pi i}\left(C_0^{EU}\right)^{1-\rho} - \left(\alpha_{\pi i} + (1-\rho)\,(1-\alpha_{\pi i})\right) K\,(Y_{eu0})^2}{\alpha_{\pi i} + \frac{\gamma}{1-\gamma}\,(\rho-1)\,(1-\alpha_{\pi i})}\right)^2 \sigma^2_{M_{us}}$$

This combination policy allows to insulate the economy from the US monetary disturbance when $\alpha_{\pi i}$ is chosen, such that the expression in brackets takes the value zero. In this case, the value of $\alpha_{\pi i}$ which produces that result must satisfy:

$$\alpha_{\pi i}\left(C_0^{EU}\right)^{1-\rho} - \left(\alpha_{\pi i} - (\rho-1)\,(1-\alpha_{\pi i})\right) K\,(Y_{eu0})^2 = 0;\ \ \alpha_{\pi i} + \frac{\gamma}{1-\gamma}\,(\rho-1)\,(1-\alpha_{\pi i}) \neq 0$$
$$\Rightarrow \alpha_{\pi i} = \frac{\rho-1}{\rho-\xi_{eu}};\quad \xi_{eu} \neq \frac{1}{\gamma}$$

[40] Under this rule: $\frac{M_{eu}}{M_{eu0}} = \left(\frac{M_{us}}{M_{us0}}\right)^{\frac{\alpha_{\pi i}+(1-\rho)(1-\alpha_{\pi i})}{\alpha_{\pi i}-\frac{\gamma}{1-\gamma}(1-\rho)(1-\alpha_{\pi i})}}$

When $\xi_{eu} = 1/\gamma$, that is, when $K\left(Y_{eu0}\right)^2 = \gamma(C_0^{EU})^{1-\rho} = \varpi^*$, for any value α the loss will be always equal to $\left(\frac{1-\gamma}{\gamma}\beta\varpi^*\right)^2 \sigma^2_{M_{us}}$.[41]

Analogously, in the case of a "*two-pillar policy with the money growth rate*" the relationship between inflation and the money growth rate is determined by $\alpha_{\pi M}$. For any $\alpha_{\pi M}$ the central bank's loss function takes the form:[42]

$$L_{M/\pi}|_{eu} = \beta^2 \left(\frac{(1-\gamma)(2\alpha_{\pi M}-1)\left(C_0^{EU}\right)^{1-\rho} - (1-\gamma)\alpha_{\pi M} K\left(Y_{eu0}\right)^2}{(2-\gamma)\alpha_{\pi M}-1} \right)^2 \sigma^2_{M_{us}}$$

In this case, the value of the parameter determining the combination that allows to insulate the economy from the US monetary shocks is the one which satisfies:

$$(1-\gamma)(2\alpha_{\pi M}-1)\left(C_0^{EU}\right)^{1-\rho} - (1-\gamma)\alpha_{\pi M} K\left(Y_{eu0}\right)^2 = 0; (2-\gamma)\alpha_{\pi M}-1 \neq 0$$
$$\Rightarrow \alpha_{\pi M} = \frac{\xi_{eu}}{2\xi_{eu}-1}; \xi_{eu} \neq \frac{1}{\gamma}$$

Recall that $\xi_{eu} = \frac{\phi}{(\phi-1)}\frac{1}{\beta(\overline{g}_{eu0})}$ and observe that this policy rule requires less information about the parameters, in particular, it requires no knowledge of the parameter of risk aversion ρ. When $\xi_{eu} = 1/\gamma$, that is, when $K\left(Y_{eu0}\right)^2 = \gamma(C_0^{EU})^{1-\rho} = \varpi^*$, the loss is, once more, equal to $\left(\frac{1-\gamma}{\gamma}\beta\varpi^*\right)^2 \sigma^2_{M_{us}}$, for any $\alpha_{\pi M}$.

For the rules using the exchange rate as a policy variable the solutions for the optimal policy parameters are obtained as in the previous cases. Under the MCI rule, for instance, for any α_{MCI}, the loss function can be written as:

$$L_{i/E}|_{eu} = \beta^2 \left(\frac{\alpha_{MCI} C_0^{1-\rho} - \left(\alpha_{MCI} + \gamma(\rho-1)(1-\alpha_{MCI}) - (\rho-1)(1-\alpha_{MCI})\right) K Y_0^2}{\alpha_{MCI} + \gamma(\rho-1)(1-\alpha_{MCI})} \right)^2 \sigma^2_{M_{us}}$$

The *MCI* policy allows to insulate the economy from the US monetary disturbance provided that the parameter α_{MCI} is chosen, such that the expression in brackets takes the value zero, which in this case occurs when α_{MCI} satisfies:[43]

$$\alpha_{MCI} C_0^{1-\rho} - \left(\alpha_{MCI} - (1-\gamma)(\rho-1)(1-\alpha_{MCI})\right) K Y_0^2 = 0;\ \ \alpha_{MCI} + \gamma(\rho-1)(1-\alpha_{MCI}) \neq 0$$
$$\Rightarrow \alpha_{MCI} = \frac{(1-\gamma)(\rho-1)}{(1+(1-\gamma)(\rho-1)) - \xi_{eu}};\ \ \xi_{eu} \neq \frac{1}{\gamma}$$

[41] Recall that $\xi_{eu} = \frac{(\overline{C}_0^{EU})^{(1-\rho)}}{k(\overline{Y}_{eu0})^2} = \frac{\phi}{(\phi-1)}\frac{1}{\beta(\overline{g}_0)}$

[42] Under this rule $\frac{M_{eu}}{M_{eu0}} = \left(\frac{M_{us}}{M_{us0}}\right)^{\frac{\alpha_{\pi M}(1-\gamma)}{\alpha_{\pi M}(2-\gamma)-1}}$.

[43] Again when $\xi_{eu} = 1/\gamma$, that is, when $K\left(Y_{eu0}\right)^2 = \gamma(C_0^{EU})^{1-\rho} = \varpi^*$, for any value α_{MCI} the loss will be always equal to $\left(\frac{1-\gamma}{\gamma}\beta\varpi^*\right)^2 \sigma^2_{M_{us}}$.

In the case of the *IMI* rule, for any α_{IMI}, "*two-pillar policy with the nominal interest rate*" the central bank's loss function can be written as:

$$L_{M/E}|_{eu} = \beta^2 \left(\frac{(2\alpha_{IMI} - 1 + \gamma(1-\alpha_{IMI}))\left(C_0^{EU}\right)^{1-\rho} - \alpha_{IMI} K \left(Y_{eu0}\right)^2}{2\alpha_{IMI} - 1} \right)^2 \sigma^2_{M_{us}}$$

The value of α_{IMI} that allows the economy to be insulated from US monetary shocks must therefore satisfy:[44]

$$(2\alpha_{IMI} - 1 + \gamma(1-\alpha_{IMI}))\, C_0^{1-\rho} - \alpha_{IMI} K Y_0^2 = 0; \quad \alpha_{IMI} \neq 1/2$$
$$\alpha_{IMI} = \frac{(1-\gamma)\xi_{eu}}{(2-\gamma)\xi_{eu} - 1}; \xi_{eu} \neq \frac{1}{\gamma}$$

A.6 Analysis of European Fiscal Shocks

This appendix shows the effects on consumption and output when (i) a fixed money supply rule or (ii) a fixed interest rate rule are used to stabilize permanent domestic fiscal shocks.

(i) Under a constant money supply rule the impact of a permanent domestic fiscal shock $g_{eu}/\overline{g}_{eu0}$ on consumption and output, in the short run and in the long run (respectively) will be equal to:

$$\begin{aligned} C^{EU} &= \overline{C}_0^{EU} \\ Y_{eu} &= \overline{Y}_{eu0} \frac{g_{eu}}{\overline{g}_{eu0}} \end{aligned}$$

$$\begin{aligned} \overline{C}^{EU} &= \overline{C}_0^{EU} \left(\frac{g_{eu}}{\overline{g}_{eu0}} \right)^{-\frac{\gamma}{1+\rho}} \\ \overline{Y}_{eu} &= \overline{Y}_{eu0} \left(\frac{g_{eu}}{\overline{g}_{eu0}} \right)^{1-\frac{(1+\gamma)+(1-\gamma)\rho}{2(1+\rho)}} \end{aligned}$$

(i) Under a constant nominal interest rate rule the impact of a permanent domestic fiscal shock $g_{eu}/\overline{g}_{eu0}$ on consumption and output, in the short run and in the long run (respectively) is given by:

$$\begin{aligned} C^{EU} &= \overline{C}_0^{EU} \left(\frac{g_{eu}}{\overline{g}_{eu0}} \right)^{-\frac{\gamma}{1+\rho}} \\ Y_{eu} &= \overline{Y}_{eu0} \left(\frac{g_{eu}}{\overline{g}_{eu0}} \right)^{\frac{\rho}{1+\rho}} \end{aligned}$$

[44] When $\xi_{eu} = 1/\gamma$, that is, when $K\left(Y_{eu0}\right)^2 = \gamma(C_0^{EU})^{1-\rho} = \varpi^*$, the loss is, once more, equal to $\left(\frac{1-\gamma}{\gamma}\beta\varpi^*\right)^2 \sigma^2_{M_{us}}$, for any α_{IMI}.

$$
\begin{aligned}
\overline{C}^{EU} &= \overline{C}^{EU} \\
\overline{Y}_{eu} &= \overline{Y}_{eu0}\left(\frac{g_{eu}}{\overline{g}_{eu0}}\right)^{\frac{\rho}{1+\rho}-\frac{(1-\gamma)(\rho-1)}{2(1+\rho)}} \\
&= \overline{Y}_{eu0}\left(\frac{g_{eu}}{\overline{g}_{eu0}}\right)^{1-\frac{(1+\gamma)+(1-\gamma)\rho}{2(1+\rho)}}
\end{aligned}
$$

Other Shocks:

Foreign Permanent Fiscal shocks: only affect the long run and monetary policy is neutral in the long run. Other shocks: the nominal interest rate reacts only to γ and ρ.

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