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No. 9 November 1982

Marginal Employment Subsidies : An Effective Policy to Generate Employment

Carl Chiarella* and Alfred Steinherr**

Internal Paper

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ABSTRACT

In this paper subsidies on employment increases above a benchmark employment level are proposed as a policy to create additional employment. For analysis a simple general equilibrium model is used with firms maximising the discounted value of their intertemporal profits and workers maximising discounted utility. Particular attention is given to the financial constraint on the subsidy programme which plays an important role for the effectiveness of the programme.

The analysis shows that marginal employment subsidies can have a significant effect on employment creation without worsening the public sector deficit. Over time also investment can be expected to increase as a response to employment subsidies. With a properly designed subsidy programme a return to "full" employment in industrial countries seems feasible within a few years even for pessimistic empirical values of the real wage elasticity of labour demand. If economic conditions worsen further in the near future this conclusion may not hold anymore in a strict sense while an upswing of the business cycle would render return to "full" employment easier.

CONTENTS

1. INTRODUCTION

Current unemployment is at unacceptably high levels for both the segment of the population concerned and policy-makers in most industrialised countries. The causes of the rise in world-wide unemployment are usually attributed to the severe supply shocks in the 1970s (the sharp increase in oil and other raw material prices), the associated redistribution of income from countries with high propensities to spend to countries with low propensities to spend, inadequate policy responses and rigidities in industrial economies responsible for slow adjustment to a different environment[1]. Rigidities seem to be more severe for the countries of the EEC than for Japan and the US and unemployment in consequence has risen more sharply in the EEC. The main rigidity can be seen in the high growth of real wage costs. Instead of attenuating the supply shocks through moderate wage increases the effects of these shocks have been accentuated in the EEC by the wage cost explosion of the 1970s. An approximate empirical measure for these effects is provided by the sharp increase of the real wage cost gap in EEC countries which, however, is an underestimate since firms have responded by reducing employment to improve productivity[2].

In such situations economies risk entering a vicious circle. High real wage growth leads to higher unemployment requiring higher unemployment compensation payments which in turn increase public sector deficits. To stem the rise in those deficits several countries have increased direct or indirect taxation, the incidence of which falls at least partly on the corporate sector (pay roll taxes and other taxes) and induces a further downward adjustment of investment and employment.

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[1] See, for example, OECD (1982) and Sachs (1979)

[2] OECD (1982) and Steinherr (1982)

The situation we face now is thus to a large extent due to the failure of the evolution of wage costs to correspond to changes in the economic environment. If this environment were to become more favourable again employment could improve automatically. However, the view is now widespread that at least for the short run no dramatic improvement of world business conditions can be expected, so that measures aimed at reducing unemployment are urgently required.

One obvious solution would be provided by real wage reductions. However, short run employment elasticities for EEC countries in the range from -0,4 to -0,8 suggest that real wage cost reductions would have to be substantial to reduce unemployment to its "natural" level[3]. Feasibility of this option is therefore severely limited and, from a distributional point of view, also quite unattractive.

Some specific proposals have been made recently such as institutional reduction of work-time per employed or a redistribution of unemployment through increased part-time employment contracts[4]. While the efficiency implications of such proposals are uncertain they also have the drawback of redistributing the incidence of unemployment rather than increasing employment possibilities. Furthermore, while these proposals are ad hoc for the current slow growth period they may create increased rigidities for a potential upswing in the future, and they may not correspond to workers preferences.

Constraints on demand policy originating from already high government budget deficits maintained for many years, combined with doubts about the effectiveness of demand stimulation even if it were feasible, make it unlikely that demand policies will serve to reduce substantially unemployment. Supporters of supply oriented policies are suggesting measures that increase profitability in the private sector and ultimately lead to increased demand for labour. Such measures comprise decreased profit taxes, accelerated depreciation allowances or other forms of investment subsidies. Most of these measures have at best a limited immediate effect on employment or only an indirect and delayed effect. If the main concern is to increase

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[3] See OECD (1982)

[4] Drèze (1979)

employment then it appears sub-optimal to subsidize another factor of production.[5] For example, reduced taxes on corporate profits affect employment only if higher profits lead to higher investments. Similarly, direct investment subsidies may not create any additional net employment, if the substitution effect dominates[6]. At any rate,since investment plans require time for realisation, employments effects if they exist will not be felt significantly in the short run[7].

Some economists and policy-makers have turned their attention to policies of direct subsidisation of employment. A general wage cost subsidy such as, for example, fiscalisation of employers' social security contribution is debated in some EEC countries[8]. Such a policy would tend to decrease the net real wage costs of firms and the effects on labour demand would therefore be comparable to a reduction in real wage rates. However, the disadvantage of a general subsidy is that the size of wage cost reduction is necessarily severely restricted, given current government budget constraints. We would also argue that general wage subsidisation is not necessary. A marginal employment subsidy (MES) has a much larger effect on marginal cost than a general subsidy costing the same amount and the impact on employment can therefore be substantial, as is perhaps most easily seen for an open economy with prices for tradable goods largely determined on world markets.

[5] This argument is also in line with the principle of optimal subsidies due to Bhagwati and Ramaswani (1963). By implication, tariff protection, as advocated by the Cambridge Economic Policy Group, would also be a sub-optimal measure for employment stimulation.

[6] Kesselman et al. (1977) estimate cost functions for US manufacturing and find that capital and white collar workers are complements, while blue collar workers and capital are substitutes. Thus investment incentives favour employment of white collar workers and adversely affect employment of blue collar workers.

[7] The empirical work by Nadiri and Rosen (1969) suggests that firms adjust labour more quickly than capital.

[8] General wage subsidies on a macroeconomic scale were first proposed by Kaldor (1936). Borts (1966) also has shown that with wage rigidities a wage subsidy is superior to output or capital subsidisation.

In the short run an MES will have the effect of stimulating hiring by the firm. This tends, under most production conditions, to increase the marginal product of capital and hence investment can also be expected to increase, after some delay. Since the MES lowers the effective marginal cost of labour relative to capital, the capital labour ratio is expected to be lower under a subsidisation scheme than without. This decline of the capital-labour ratio is not, however, necessarily a distortion. In fact, it may correct existing distortions which result from capital subsidisation schemes in application[9].

Since hiring and investing usually involves frictional costs in addition to rental costs, one would expect that a short-run subsidy programme would be less efficient than a longer-run programme. Adjustment costs and the duration of the programme (or the likelihood of its maintenance) are therefore essential elements in the analysis of such a programme.

Administration of MES raises, of course, some practical difficulties. While subsidies could be made specific for certain skill groups or employment in particularly depressed areas, we shall consider in this paper an MES provided for the entire net increase in employment, taking as benchmark employment at a given time before announcement of the subsidy programme. This implies, of course, that jobs that would have been created without the subsidy would also receive it. If net employment creation in all industries is small (or negative) this aspect can be neglected. On the other hand, if employment creation is important in some sectors of the economy then it might be preferable to limit the subsidy to sectors where employment is declining. At any rate the subsidisation scheme should be kept administratively as simple and neutral as possible in order not to replicate the inefficiency, opaqueness and contradictions of existing investment subsidisation schemes.

A future practical issue is the time horizon of the subsidy programme. If the programme is very short-lived then neither employment nor investment will even temporarily be much stimulated when there are adjustment costs in addition to rental payments. How long-lived the

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[9] For an international comparison of capital subsidies see Kopits (1980).

programme ideally should be depends essentially on the time profile of the general business cycle. If the economic environment remained stationary forever (but real wages still remained above the level consistent with full employment !) then the employment subsidy might best be permanent. Of course, given the skill-structure of the unemployed and the skills demanded by firms it might not be possible with any general, as opposed to skill specific, policy to create jobs for all. The target for the subsidy policy is therefore to reduce unemployment to a level considered as "normal".

Layard and Nickell (1980) analyse MES within a Keynesian framework. Their main conclusion is that the budget deficit cost per additional job is less for the MES than for an increase in government expenditure, and that the balance of payments and price level effects are always more favourable. Holmlund (1978) uses an inter-temporal maximisation model for the competitive firm and alternatively for a dynamic monopsony in the labour market. He shows that a pure hiring subsidy always increases the equilibrium capital and employment levels.

Both papers fail to impose a financial constraint on the subsidisation policy. Layard and Nickell follow Keynesian procedures allowing government budget deficits to increase without any feedback on agents' expectations about future tax liabilities. Holmlund also neglects any government budget constraint which is justified by his focus on a single firm. The subsidy he analyses is also a very special one where firms receive once and for all a certain amount per hiring.

The approach taken in this paper uses an intertemporal maximisation model for a representative firm. Subsidies are paid for each additional hiring forever, that is each period, but there is an increasing probability over time that the programme will be discontinued. Furthermore, we introduce finance constraints on the MES and the representative firm takes into account its share of future tax liabilities. This essentially makes it more difficult for the subsidy to stimulate employment than in a partial equilibrium framework or in the Keynesian framework without a budget constraint.

Section 2 serves to lay foundations and presents the model. Section 3 contains an analysis of the MES under the constraint that the initial government budget deficit cannot be exceeded. Section 4 repeats the analysis under a more general constraint where government finances deficits on external capital markets and where the cost of borrowing is covered through taxes on firms. The analysis of Sections 3 and 4 is focused on the open sector of the economy which represents more than half of total employment in EEC economies and on which fell the brunt of the decline in employment. To complete the investigation and to verify that employment creation is not achieved simply through exporting unemployment. Section 5 analyses a closed economy. Section 6 summarises the main conclusions of the paper. This paper is mainly theoretical and notes the practical implications only in passing. A second paper (in preparation) discusses in greater detail the practical aspects of policy implementation and surveys the available empirical evidence.

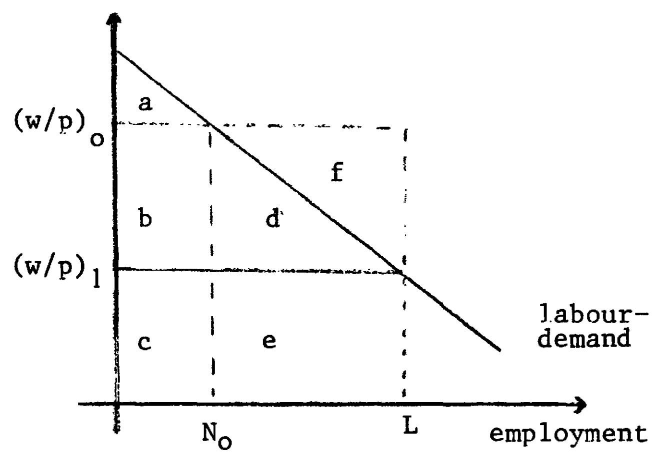
2. THE FRAMEWORK

We first present the basic idea motivating this paper in a static framework and then describe the model used for the analysis to follow.

2.1 Marginal employment subsidies in a static labour market

To facilitate the analysis of MES we compare such subsidies with a reduction in real wages in the simplest possible framework,the standard static text-book representation of the labour market. We therefore postulate a macroeconomic demand function for labour which is assumed to be a non-negative function of the real wage rate. The capital stock, tax structure and monetary conditions are assumed constant. In line with actual practice in EEC countries the full employment target (L) is considered to be exogenously given as the sum of current employment (N_o) plus unemployment minus a correction for "normal" unemployment. Furthermore the real wage rate (w/p) is taken as being fixed by labour unions. These are stark assumptions but they capture the essential features of the problem. A recapitulation is contained in Figure 1.

Figure 1



At the initial employment level N_o gross domestic product (GDP) is equal to the surface (a + b + c), with (b + c) representing labour income and (a) non-labour income. A real wage reduction to $(w/p)_1$ would create full employment and a higher GDP. However, labour income, now equal to (c + e), would fall in absolute terms with an inelastic labour demand curve and non-labour income would very strongly increase absolutely and as a proportion of GDP.

Consider now an MES for employment above N_o, keeping real wages constant. For labour demand to rise to full employment the unit subsidy must equal $\{\frac{W}{P}\}_o - \{\frac{W}{P}\}_1$. Employment and GDP are as with a wage reduction, but labour now receives (b + c + d + e + f) and non-labour income is equal to (a + d). The area (d + f) represents the employment subsidy. Since GDP is only (a + b + c + e) gross factor incomes exceeds net factor incomes by (d + f), the amount of taxes required to finance the subsidy programme. But it is to be noted that also in initial equilibrium gross incomes are taxed to finance compensation for the unemployed. Suppose therefore that unemployment compensation is financed with taxes levied on non-labour income and, furthermore, that unemployment compensation initially does not fall short of the area (d + f). Then two conclusions follow : (i) the employment subsidy is feasible even under the constraint that the absolute tax burden on non-labour income should not increase (the tax relative to non-labour income would of course decline) ; (ii) the share of non-labour income in GDP would rise since a/(b + c) is smaller than d/e.

Thus, with a real wage cut labour income falls and non-labour income rises substantially whereas with an MES labour income rises in proportion to the employment increase while only the share of labour income falls. If the tax incidence fell on labour income rather than on non-labour income the analysis and the results would remain unchanged. Differences would only arise if the tax incidence of unemployment payments were opposite to the one of employment subsidies.

Essentially two factors differentiate a wage-cut policy from MES. First, with a wage-cut policy non-labour income not only gains (b + d) but also saves taxes which finance the unemployed in the initial equilibrium. By contrast, under the subsidy scheme non-labour income would still be taxed in the new equilibrium to finance subsidies. Second, MES extracts the intra-marginal revenue (b) from non-labour income. That is, labour and government together act like a monopolist "riding down" the demand curve which is of course the optimal strategy for any monopolist when discrimination is feasible.

The questions we wish to analyse in this paper, for a given MES, is as follows.

(i) Is employment necessarily rising ?

(ii) Can full employment be reached under different constraints on subsidy payments ?

(iii) What are the characteristics of the time path from initial employment to the new equilibrium employment ?

(iv) How are the results affected when the duration of the subsidy programme is either finite or stochastic ? How do risk-attitudes of employers affect the results ?

(v) Is there an optimal subsidy programme ?

(vi) Do the conclusions depend on whether the economy is open or closed or, in other terms, do the conclusions depend on the degree of openess of the economy ?

Analysis of these questions requires a dynamic general equilibrium framework. To keep the analysis manageable we specify an extremely simplified model which captures however the features of interest to us.

2.2 The Model

We consider an economy with n identical firms. In the open economy firms are price-takers in their output market with the price given by the world market. They are also price-takers in factor markets, the price of the capital good being m and the wage cost being w. The latter can be viewed as being imposed by labour unions. Firms have a production function Q(K, N) with non-increasing returns to scale, where K is the capital stock and N the employment level of the firm at time t. We also assume that investment (I) and hiring (A) give rise to adjustment costs of the following form[10]:

(1) $$C(A, I) = \frac{a_o}{2} A^2 + \frac{a_1}{2} I^2, \text{ with } a_o, a_1 > 0.$$

The firm is assumed to maximise the present value of its profits stream net of taxes (T) and subsidies (S). We consider an MES of the form :

(2) $$S = s(N - N_o)$$

(10) The assumption of convex adjustment costs in A and I is standard in the literature. See for example Gould (1968).

The subsidisation scheme (2) implies that any net hiring above initial employment N_o is subsidised at the rate s per worker as long as the policy is in affect. Holmlund analyses an MES of the form $S = s(A - kN)$ which amounts to subsidising new employment only at the moment of hiring. For $k > 0$ only hiring above the threshold level kN is subsidised. We shall show that for $k = 0$ the two schemes for MES are technically equivalent, but not in their incentive effects.

In general, a firm cannot be certain that the subsidy will be provided forever. We postulate that at $t = 0$ there is a probability path $\sigma(t)$ that the subsidy will be maintained at any moment in time $t > 0$. We assume that this probability declines over time as follows :

(3) $$\sigma(t) = \sigma_o + \sigma_1 e^{-\varepsilon t}, \text{ with } \varepsilon > 0 \text{ ; } 0 < \sigma_o, \sigma_1 < \sigma_o + \sigma_1 \leq 1 .$$

The firm seeks hiring policy A and investment I so as to maximise the discounted stream of future profits (π_t, $t \geq 0$) :

(4) $$V = \int_o^\infty e^{-rt} \pi^\gamma \, dt$$

where

(5) $$\pi = pQ(K, N) - wN - C(A, I) - mI + \sigma S,$$

subject to the constraints

(6) $$\dot{N} = A - qN$$

(7) $$\dot{K} = I - \delta K,$$

and a constraint on the public sector deficit to be specified in Sections 2 and 3. In (4) r is the exogenous discount rate and γ a measure of risk aversion ($o < \gamma \leq 1$) ; in (6) and (7) q is the quit rate and δ the rate of capital depreciation. Definition (4) is an approximation of the discounted utility from the expected profit stream. In a deterministic context the maximisation problem can be given the following reinterpretation : $\sigma(t)s$ represents a pre-announced path for the subsidy rate and firms maximise the concave utility function π^γ.

3. EMPLOYMENT SUBSIDIES CONSTRAINED BY INITIAL UNEMPLOYMENT COMPENSATION EXPENDITURES

We consider as starting point for our analysis a situation close to one which can be observed in many economies : high levels of unemployment where the unemployed receive unemployment benefits expressed as a percentage of current wages for the employed. Given also the state of public finance in those economies any new policy ideally ought to respect the constraint of not increasing further the government's budget deficit. In this section we impose therefore this constraint in the following form : employment subsidies at any point in time are not allowed to exceed the savings in disbursements of unemployment benefits. With n identical firms this constraint can be written :

$$(8) \qquad ns(N - N_o) + w\alpha(L - nN) \leq w\alpha(L - nN_o)$$

or,

$$(8') \qquad s \leq w\alpha,$$

where nN_o is the initial employment level for the economy, L is the exogenously given full employment level, N is current employment, w is the exogenous nominal wage rate, and α is the proportion of wages provided as unemployment benefits. The LHS of (8) is the sum of subsidies and unemployment benefits at time t which must not exceed initial unemployment benefits on the RHS. This constraint collapses to (8') which says simply that the subsidy per worker must not exceed the unemployment benefit per unemployed. Constraint (8') will actually not affect the optimisation of the firm but only the intervention of the policymaker.

The objective function in (4) being concave in the state and control variables we only are concerned with the necessary conditions for an optimal solution. Application of standard optimal control theory techniques[11] yields the necessary conditions :

$$(9) \qquad \lambda_1 = C_A \pi^{\gamma-1} \gamma$$

[11] See, for example, Arrow and Kurz (1970).

(10) $\lambda_2 = (C_I + m)\pi^{\gamma-1}\gamma$

where λ_1 is the present value of the shadow-price for labour, and λ_2 the present value of the shadow-price for capital. λ_1 and λ_2 are determined by the differential equations

(11) $\dot{\lambda}_1 - (r + q)\lambda_1 = (-\ pQ_N + w - \sigma s)\pi^{\gamma-1}\gamma$

(12) $\dot{\lambda}_2 - (r + \delta)\lambda_2 = -pQ_K\pi^{\gamma-1}\gamma$.

From (9) to (12) one obtains :

(13) $\pi^{\gamma-1}(t)\ C_A = \int_t^\infty e^{-(r + q)\ (v - t)}(pQ_N - w + \sigma s)\ \pi(v)^{\gamma-1}dv$

(14) $\pi^{\gamma-1}(t)\ (C_I + m) = \int_t^\infty e^{-(r + q)\ (v - t)}(qQ_K)\pi(v)^{\gamma-1}dv.$

Equation (13) says simply that at each instant labour is hired up to the point where the instantaneous marginal adjustment cost, converted to utility losses ($\pi^{\gamma-1}(t)C_A$) just balances the discounted utility gains over all future time arising from the additional unit of employment. Equation (14) has a similar interpretation for investment.

3.1 Equilibrium solution

The equilibrium solution is characterised by[12] :

(15) $pQ_N = (r + q)\ C_A + w - \sigma_o s$

(16) $pQ_K = (r + \delta)\ (C_I + m)$.

[12] If an MES of the form $S = s(A - kN)$ is used then condition (15) becomes (15') $pQ_N = (r + q)\ C_A + w + (k - r - q)s$ and (16) remains unchanged. For s to have a positive effect on employment the condition $k < r + q$ must be met. If $k = 0$ then (15) and (15') are identical since a subsidy of one dollar paid once and for all at the time of hiring is equal to a continuous subsidy stream of $(r + q)$ dollars. However, when the once and for all subsidy is received there is no incentive to maintain employment when $k = 0$. On the other hand, for $k > 0$ (15') implies a lower effect of subsidies on employment than (15).

These conditions show that risk attitudes do not affect the equilibrium solution, only the dynamic paths. From (15) we see that the subsidy lowers the effective marginal wage cost and thus an increase in the expected subsidy has the same effect as a decrease in the wage rate. Differentiating totally (15) and (16) yields :

$$(17)\qquad pQ_{NN}dN + pQ_{NK}dK = (r + q)\ C_{AA}qdN + d(w - \sigma_o s)$$

$$(18)\qquad pQ_{KN}dN + pQ_{KK}dK = (r + \delta)\ C_{II}\ \delta dK.$$

Using $C_{AA} = a_o$ and $C_{II} = a_1$ we obtain :

$$(17')\qquad \{pQ_{NN} - (r + q)a_o q\}dN + pQ_{NK}dK = d(w - \sigma_o s)$$

$$(18')\qquad pQ_{KN}dN + \{pQ_{KK} - (r + \delta)a_1\delta\ \} = 0\ .$$

Hence

$$(19)\qquad \frac{dN}{d(w - \sigma_o s)} = \frac{1}{\Delta}\ \{pQ_{KK} - (r + \delta)a_1\} < 0$$

$$(20)\qquad \frac{dK}{d(w - \sigma_o s)} = \frac{1}{\Delta}\ (-pQ_{NK}) < 0\ ;$$

where

$$\Delta = \{pQ_{KK} - (r + d)a_1\delta\}\{pQ_{NN} - (r + q)a_o q\} - p^2 Q_{NK}{}^2 > 0$$

(assumed)[13].

Thus we see from (19) and (20) that an increase in employment subsidies increases both equilibrium employment and the equilibrium capital stock.

A question of considerable interest is whether any feasible subsidy will steer the economy to full employment. We consider the

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[13] $Q_{KK}Q_{NN} > Q_{NK}{}^2$ is a standard condition, which always holds for a Cobb-Douglas production function with non-increasing returns to scale. Adjustment costs only strengthen this condition.

least favorable case where the capital stock remains constant : if full employment can be reached with this condition then it is also reachable and even much more easily when capital adjusts. With K fixed (19) can be rewritten as :

$$(19') \qquad \frac{dN}{d(w - \sigma_o s)} = \frac{1}{pQ_{NN} - (r + q)a_o q} .$$

Defining η as the elasticity of employment with respect to net wages we obtain

$$(19'') \qquad \eta = \frac{dN}{d(w - \sigma_o s)} \frac{(w - \sigma_o s)}{N} = \frac{1}{pQ_{NN} - (r + q)a_o q} \frac{w - \sigma_o s}{N} < 0.$$

Hence

$$(20) \qquad \frac{dN}{N} = \eta \frac{d(w - \sigma_o s)}{w - \sigma_o s} > 0 \text{ for } d(w - \sigma_o s) < 0.$$

From constraint (8) we need $\sigma_o s \leq w\,\alpha$. With wages fixed, $d(w - \sigma_o s) = - \sigma_o\, ds$ and if in the initial equilibrium $s = 0$ then $\sigma_o\, ds \leq w\,\alpha$. Hence

$$\frac{dN}{N} = - \eta \frac{\sigma_o}{w} ds \leq -\eta\alpha, \text{ or}$$

$$(20') \qquad \frac{1}{\alpha} \frac{dN}{N} \leq - \eta .$$

In European economies α ranges typically between 0,5 - 0,8 for the short-run (one year), while the unemployment rate is around 0,10. With a target unemployment rate of 0,05 the most difficult policy case is therefore for $\alpha = 0{,}5$ and a desired increase in employment by 0,05. To reach this target, condition (20') requires that $- \eta \geq 0{,}1$. If in addition, the subsidy were only provided to the open sector of the economy then the share of the latter in total employment becomes important. Taking 0,5 as a lower bound for this share the labour demand elasticity would have to exceed 0,2 to generate an increase of employment in the open sector by 10 percent, equal to a 5 percent increase of economy-wide employment. Except for the very short-run (less than a year) $- \eta = 0{,}2$ can safely be considered as a lower bound on empirical labour demand elasticities so that employment subsidies should lead the economy to full employment over the short to medium run.

3.2 Dynamics

The eigenvalues for the subsystem (N, A) are (see equation (4) in the Technical Appendix) :

(21) $$\rho_1, \rho_2 = \frac{r}{2} \pm \frac{1}{2} \Delta$$

where

$$\Delta^2 = r^2 + 4 \{q(r + q)a_o + p|Q_{NN}|\}/\{\alpha_o + (1 - \gamma)a_o^2 q^2 N^2/\pi\}$$

and with all quantities evaluated at equilibrium.

Clearly, $\rho_1 > 0$ and $\rho_2 < 0$ so that the equilibrium displays locally saddle-point behaviour with the stable arms of the saddle forming the unique optimal path. The absolute value of the negative root p_2 is the speed with which the optimal path locally approaches equilibrium.

It is now of interest to determine how parametric changes, in particular of γ, σ, s affect the speed of adjustment. The derivatives of ρ_2 with respect to γ, σ and s are given by (5) in the Technical Appendix. They have the following signs :

(22) $$\frac{d|\rho_2|}{d\gamma} > 0, \quad \frac{d|\rho_2|}{d\sigma} < 0, \quad \frac{d|\rho_2|}{ds} < 0 .$$

Increased risk aversion as measured by a decline of γ leads to a lower speed of adjustment. Thus, whereas final equilibrium is independent of the degree of risk aversion dynamic adjustment does depend on risk attitudes. The analysis confirms the intuition that risk aversion creates a lower employment response to wage subsidies along the adjustment paths.

While the equilibrium analysis has shown that an increase in σs generates a higher equilibrium employment level, such an increase tends to reduce the speed of adjustment to equilibrium. The reason for this trade-off pattern will become clearer in the discussion of the impact effects to which we now turn.

The optimal time paths for N and A obtained by linearisation around the equilibrium $(\bar{N}, \bar{A})$ are given by equations (8) and (9) in the

Technical Appendix and are reproduced as :

(23) $N(t) = \bar{N} + (N_o - \bar{N}) \exp(\rho_2 t)$

(24) $A(t) = \bar{A} + (N_o - \bar{N})(q + \rho_2) \exp(\rho_2 t)$.

We first consider a change in γ and its effect on the time path. As γ increases ρ_2 increases in absolute value. This implies that for any t, employment N(t) is increased. The effect on A is more complex : for t = 0,A jumps to a higher value A(0) and moves at a higher speed. Given that in final equilibrium A is independent of γ this implies that eventually the time path under less risk aversion is closer to equilibrium for any value $t \geq t$. These facts are depicted in Figures 2a and 2b for the case $N_o < \bar{N}$ which concerns us.

From (23) and (24) it can be seen that a change in γ leaves N(0) unaffected but A(0) shifts in proportion to γ, that is, lower risk aversion implies a stronger initial hiring response. However, since final equilibrium is independent of γ the hiring response after some time is bound to fall below the one obtained with higher risk aversion.

We now turn to analysing the effect of changes in σs on the dynamic path. These effects are most clearly seen by viewing the dynamic optimal path in the (N, A) phase-plane. This path is labelled PP in Figure 3a,passes through the point $(\bar{N}, \bar{A})$ lying on the line $\dot{N} = 0$ and has slope (see equation (11) of the Technical Appendix) :

(25) $$\frac{dN}{dA} = \frac{1}{q + \rho_2} .$$

The effect of an increase in σs is to move $\bar{N}$ and $\bar{A}$ upward along the $\dot{N} = 0$ curve. Thus the optimal path PP moves to the right as depicted in Figure 3b.

It is clear from Figure 3b that the stronger the increase in σs the larger will be the jump in hiring at time 0. It is easy to verify that a decrease in σ decreases ρ_2 in absolute value and hence increases the slope of the adjustment path p^*p^*. Therefore the initial jump in hiring will be smaller with higher risk aversion.

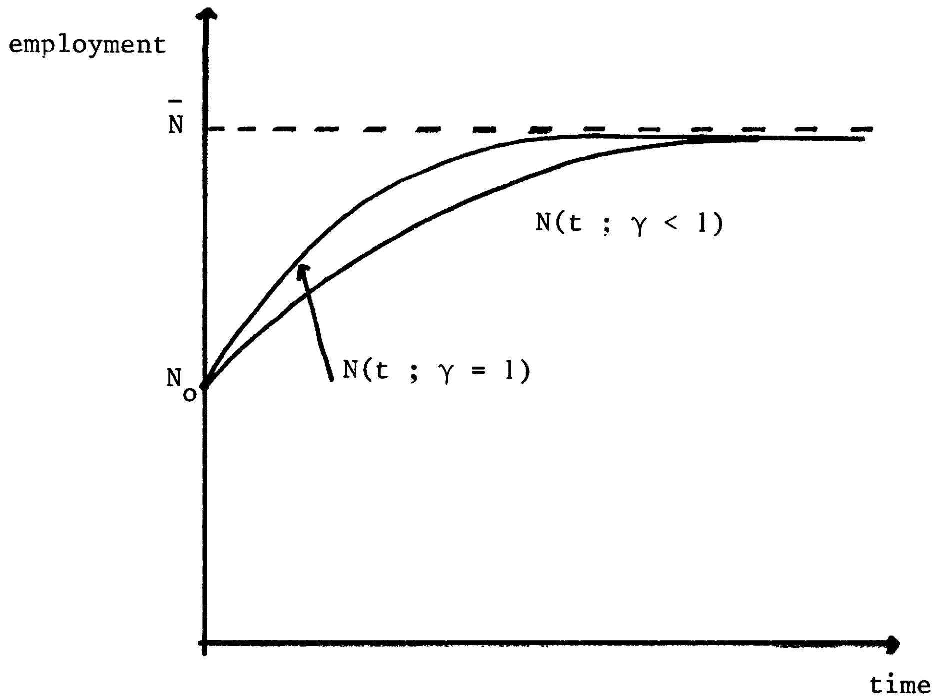

Figure 2(a)

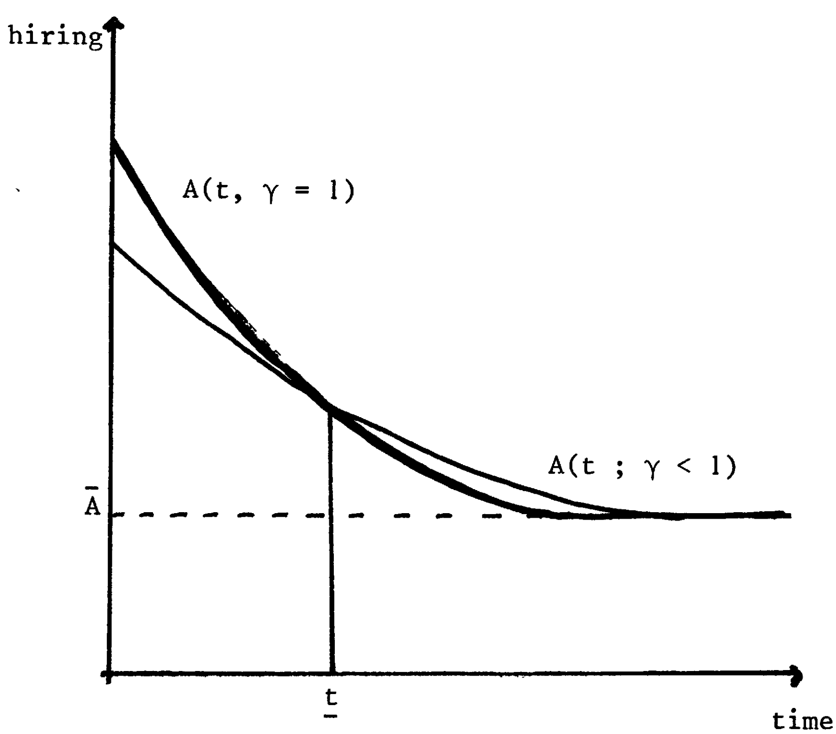

Figure 2(b)

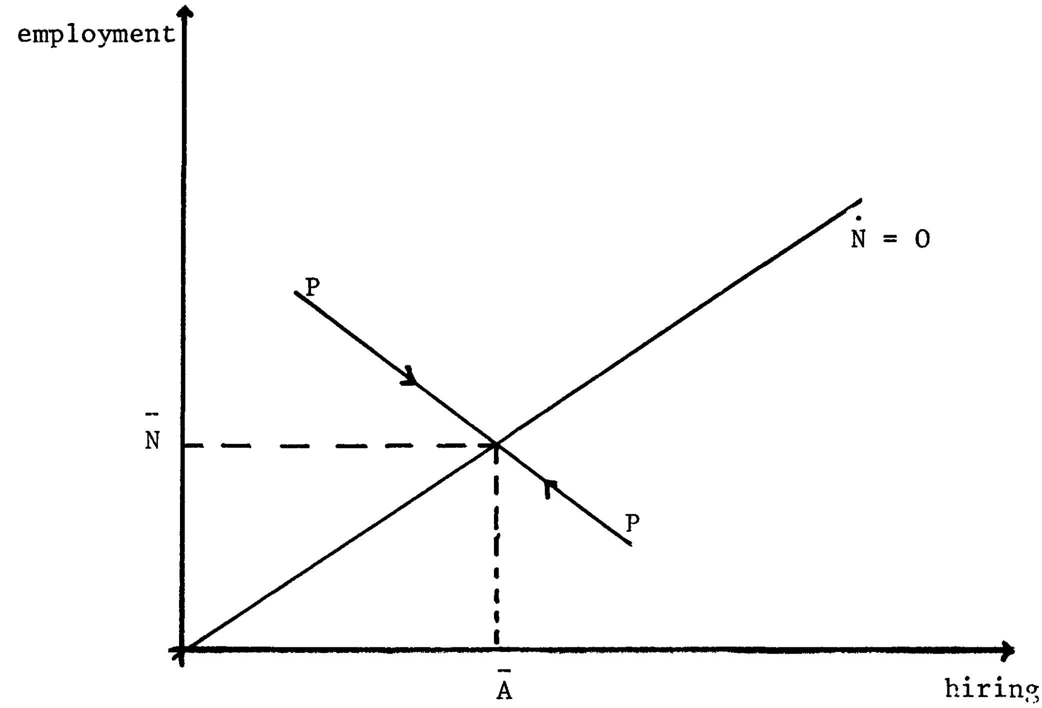

Figure 3(a)

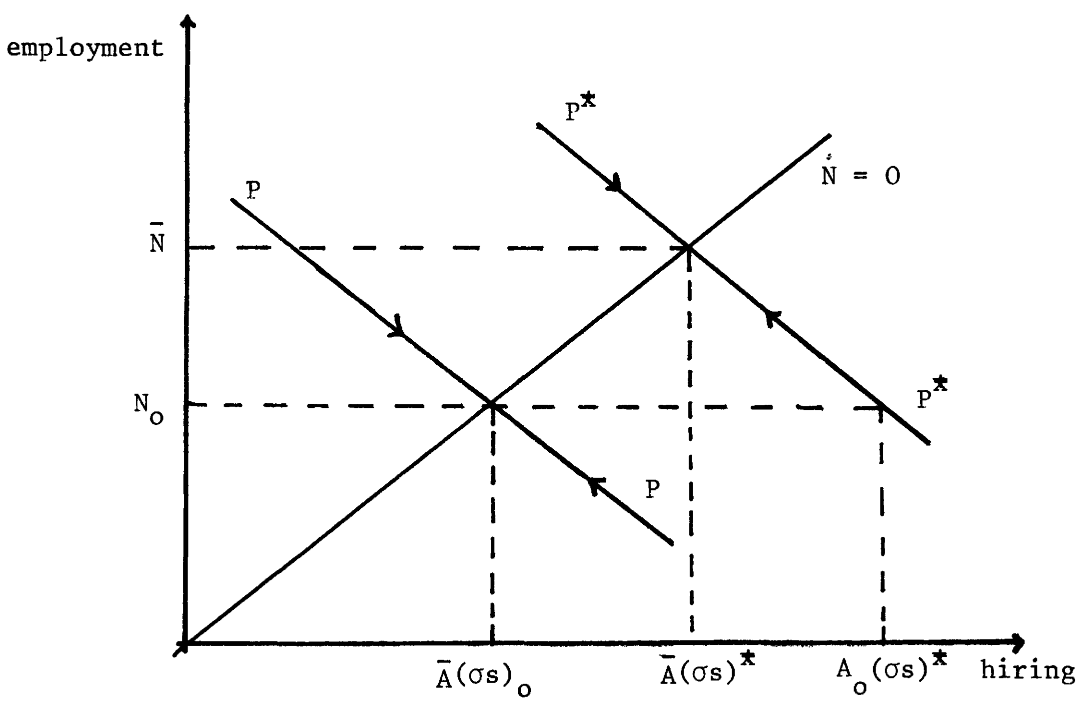

Figure 3(b)

4. FISCAL DEFICITS FINANCED ON EXTERNAL CAPITAL MARKETS

In Section 3 an arbitrary, albeit empirically highly relevant constraint was imposed on the availability of funds for employment subsidies. In this section a more general constraint is introduced. More precisely, the government is assumed to be able to finance any fiscal deficit by borrowing on international capital markets at a given interest rate r. Furthermore it is assumed that loans have to be repaid at the rate f. The evolution of the foreign debt (F) can then be written :

$$(26) \qquad \dot{F} = n\sigma s(N - N_o) + w\alpha(L - nN) - fF.$$

The cost of foreign borrowing, consisting of interest payment and repayment of the principal is supposed to be borne by firms. This assumption is made in order not to create a fiscal deficit whose financing occurs outside of the model. Of course, one could also envisage the case where consumers or workers are taxed in order to cover the cost of foreign borrowing but this raises the question whether unions base their wage claims on gross wages or on wages after taxes. In the latter case a tax on wages would simply be rolled over to a higher real wage rate. In the former case one could, of course, envisage that unit wage costs remain constant but that real income of wage earners declines as a consequence of the tax.

We specify therefore several hypotheses concerning financing of the foreign debt. The first hypothesis is that the tax is imposed on firms where the tax on the representative firm equals :

$$(27) \qquad T = (r + f)F/n .$$

Equations (26) and (27) imply that the firm takes into account the fact that the other n - 1 firms behave in exactly the same way as itself. The second hypothesis corresponds to the case where firms behave only according to price information, i.e., as a competitive firm. Then (26) has to be rewritten :

$$(26') \qquad \dot{F} = \sigma s(N - N_o) + w\alpha(\frac{L}{n} - N) - fF .$$

(26') has the following interpretation. The firm calculates its share for the evolution of the foreign debt. When it considers a change in its employment level it only takes into account the effect of its own change in employment on foreign debt and not the fact that (n - 1) other firms may do exactly the same.

The third hypothesis embodies the possibility that taxes fall on other agents than the firm. For example, they are imposed on wage earnings and for definiteness and simplicity we assume, in addition, that wages do not change in consequence.

We now analyse these three cases in turn.

4.1 Analysis of three cases of tax incidence

Case 1

Here profits are redefined as :

(28) $$\pi = pQ(K, N) - wN - C(A, I) - mI + \sigma s(N - N_o) - T \ ,$$

where T is defined in (27) .

Maximisation of $\int_o^\infty e^{-rt}\pi^\alpha dt$ subject to

(6) $$\dot{N} = A - qN$$

(7) $$\dot{K} = I - \delta K$$

(26) $$\dot{F} = n\sigma s(N - N_o) + w\alpha(L - nN) - fF$$

yields the first-order conditions :

(29) $$\lambda_1 = \gamma\pi^{\gamma-1}C_A$$

(30) $$\lambda_3 = \gamma\pi^{\gamma-1}(C_I + m)$$

and the differential equations :

(31) $\dot{\lambda}_1 - (r + q)\lambda_1 = - \{\gamma\pi^{\gamma-1}\pi_N + \lambda_2 n(\sigma s - w\alpha)\}$

(32) $\dot{\lambda}_2 - (r + f)\lambda_2 = \gamma\pi^{\gamma-1}(r + f)/n$

(33) $\dot{\lambda}_3 - (r + \delta)\lambda_3 = - \gamma\pi^{\gamma-1}pQ_K.$

Equilibrium values are determined by :

(34) $pQ_N = (r + q)C_A + (1 - \alpha)w$

(35) $pQ_K = (r + \delta)(C_I + m)$

(36) $F = \frac{1}{f}\{n\sigma_o s(N - N_o) + w\alpha(L - nN)\}.$

As can be seen from (34) the subsidy has no effect on final equilibrium employment. In the Technical Appendix the solutions for N(t), A(t) and F(t) (equations (19) - (21)) show that, in fact, employment and foreign borrowing always remain at their initial equilibrium levels. The reason for the inefficacy of the subsidy is quite simple : as can be seen from (36) firms in initial equilibrium balance the marginal product of labour with its marginal cost, from which the savings of tax liabilities due to a unit decrease in unemployment are deducted. When they are offered a subsidy they realise that over time they have to pay for it to cover the cost of borrowing : the present value of a unit subsidy exactly equals the present value of the increase in future taxes. The inefficacy of the subsidy is therefore due to the combination of a severe finance constraint with the fact that each firm knows that all other firms behave identically. This assumption corresponds therefore more closely to a monopolised than to a competitive market. The next case considers a competitive market.

<u>Case 2</u>

Here we maximise as in case 1 but replace constraint (26) by (26'). From similar calculations the equilibrium conditions are given by :

(37) $$pQ_N = (r + q)C_A + (1 - \frac{\alpha}{n})w - (1 - \frac{1}{n})\sigma_o s$$

(38) $$pQ_K = (r + \delta)\ (C_I + m)$$

(39) $$F = \frac{1}{f}\ \sigma_o s(N - N_o) + w\alpha(\frac{L}{n} - N)\ .$$

The implication of (39) is that while foreign debt is nf the representative firm only perceives the foreign debt due to its own contribution. Condition (37) collapses to the result obtained in case 1 if $n = 1$: the subsidy has no effect on equilibrium employment. By contrast if n were very large then the equilibrium result is identical to the one obtained in Section 3. The reason why the subsidy affects equilibrium employment and capital stock for $N > 1$ is the following. When the firm hires an additional worker it receives in equilibrium an expected subsidy $\sigma_o s$. Tax liabilities increase by $\sigma_o s - w\alpha$ but the firm only has to pay $\frac{1}{n}(\sigma_o s - w\alpha)$.

In the Technical Appendix a dynamic analysis similar to the one in Section 3 is carried out. For $\gamma < 1$ the analysis is intractable so we focus on the special case $\gamma = 1$. For $\gamma = 1$ the eigenvalues of the differential equation system (N, A, F, λ_3) are :

(40) $$\rho_1 = -f,\ \rho_2 = \frac{r - \Delta}{2}\ ,\ \rho_3 = r + f,\ \ \rho_4 = \frac{r + \Delta}{2}$$

where Δ is as defined in (21). Clearly, for $f > 0$ the equilibrium has the desired local saddle-point property[14]. As shown by equations (19) - (21) in the Technical Appendix the speed of adjustment of N and A is determined only by ρ_2, while the adjustment of F and λ_3 depends on both ρ_1 and ρ_2. Hence the dynamic paths for N and A have the same qualitative properties with respect to changes in σs as those analysed in Section 3.

[14]The case $f = 0$ gives rise to a different analysis since N would then be directly determined from constraint (26) and independently of the firm's optimising behaviour.

Case 3

If the incidence of the cost of foreign debt falls on wage income and w remains constant then neither constraint (26) nor tax liabilities exist anymore for the firm. The case is then identical to the one analysed in Section 3 with the only difference that the upper limit on the subsidy payments exists no longer.

4.2 Optimal Employment Subsidies

So far the subsidy rate s has been treated as arbitrarily fixed by government and we have analysed the effects of variations in that rate. We now consider the question whether there is, in fact, an optimal rate s by which we mean the following. If the government offered to firms the choice of s, under the various constraints layed out previously, does there exist a value of s that maximises firms' profits ? Technically, s becomes an additional control variable in the firm's optimisation problem.

In the analysis of case 1 we have seen that the subsidy has no effect on the system, therefore the question whether there exists an optimal value s is meaningless.

As we have already noted before when n is large case 2 and the case considered in Section 3 have the common feature that in final equilibrium employment depends positively on the value of s. This means that government in its pursuit of full employment will always set its optimal s as high as permitted by the constraints, as long as s does not exceed the value required for full employment.

If the firm is to choose its preferred s, will it similarly choose s as high as possible so that we can be confident that as much employment as possible will be created in full harmony between individual maximising behaviour and the societal goal of achieving full employment ?

The analysis is relegated to the Technical Appendix. There we show that in equilibrium firms wish to set s as high as possible (as the government would have wished to set s), so that the maximal effect on equilibrium employment is assured. While we have not undertaken an analysis of the time path of optimal subsidy the set-up of the maximisation problem suggests the conjecture that s will move as rapidly as possible to its upper bound[15].

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[15] A bang-bang solution cannot occur due to the presence of adjustment costs and risk aversion.

5. MARGINAL EMPLOYMENT SUBSIDIES IN THE CLOSED ECONOMY

We now analyse MES in an economy which does not exchange goods with the rest of the world. By definition, this is the case for the non-traded sector of an economy. Furthermore, no international capital transactions can take place and we assume that the nominal rate of interest is maintained equal to the rate of time preference r. However, in the present model prices become endogenous and therefore also the real rates of interest $r - \dot{p}/p$.

The framework we now use can be described as follows. For any time paths of prices and subsidies, maximisation of real profits by firms yields a supply schedule for goods or, equivalently, a demand schedule for investment. Similarly, for any time paths of prices and subsidies utility maximisation by workers yields a demand schedule for goods or equivalently, a supply schedule for savings. Equilibrium in the goods market requires aggregate investment to equal aggregate savings. This equilibrium condition can be solved at any time t for the equilibrium price p as a function of s.

The analysis is relegated to the Technical Appendix. Before discussing the results some additional features of the model will be described. The real wage remains fixed in this model but, to close the system, it is assumed that subsidies are paid with taxes imposed on workers. Expected income for any worker can then be derived as follows. We first assume that the probability of being unemployed is uniform for each member of the labour force and for each t : $\omega = N/L$ is the probability of being employed and $(1 - \omega)$ is the probability of being unemployed. If a worker is unemployed he receives a real unemployment compensation αw ; if he is employed he receives a real salary w. Expected real income of a member of the labour force is then

$$(41) \qquad y = \omega w + (1 - \omega)\alpha w - s(N - N_o)/L + nrV/pL$$

$$= \frac{1}{L}\{wN + \alpha w(L - N) - s(N - N_o) + nrV/p\}$$

where unemployment compensation and subsidies are assumed adjusted for inflation. Capital accumulation is financed with workers' savings and the monetary value of a firm's capital stock is V. Expected income identically equals the sum of expected consumption and savings. Aggregation over the labour force yields :

(42) $$L(E + \frac{\dot{W}}{p}) = wN + \alpha w(L - N) - s(N - N_o) + \frac{nrV}{p}$$

where W = nV/L, and E and $\dot{W}/p$ are, respectively, real consumption and real savings. Savings can only be invested in firms so that :

(43) $$\frac{n\dot{V}}{p} = nI = \frac{L\dot{W}}{p}$$

Thus, for any given price path firms are constrained by effective demand of the labour force. If for any p investment demand exceeds savings the price has to fall, inducing an increase in the real rate of interest and therefore a decline in desired aggregate investment and an increase in desired aggregate savings. For example, when an MES is offered to firms they tend to respond by hiring more labour and install more capital to offer a higher output. To make this possible workers' savings have to increase. The real rate of interest therefore must rise implying that prices must fall.

The formal model is set up in the Technical Appendix and yields exactly the same equilibrium conditions for employment and the capital stock as in equations (15) and (16) of Section 3. Hence, the effects of a MES on equilibrium employment and capital stock in a closed economy are identical to those in an open economy. The adjustment path is however likely to be different.

6. CONCLUSIONS

In this paper we analyse MES as a policy to generate more employment. Unemployment in this paper is viewed as a disequilibrium feature caused by rigid real wages rather than as an equilibrium phenomenon in the spirit of Lucas (1981). For definiteness, the economic environment is taken as stationary and we determine the effects of subsidies in that stationary environment and characterise the dynamic path and the new equilibrium solution. If, in reality, the environment were to improve in the future the subsidy could of course be partially or totally discontinued.

Economic structure is often usefully characterised in terms of traded (or open) and non-traded (or closed) sectors. We analyse both cases in turn, assuming that the entire economy is open or, alternatively, closed. The results we think are close approximations of the case where one or the other sector receives the subsidy since casual empirical observation suggests that the actual resource flows between sectors is constrained by institutional and behavioural rigidities. Since the results in both cases are qualitatively identical the overall effects of subsidies provided to both sectors are approximated by their sum.

We find that the effectiveness of an MES depends on a variety of factors. First, the effects depend on the base on which the subsidy operates. Second, a transitory subsidy will only have transitory effects on employment. Third, the effectivenss of the subsidy depends on how it is financed. In the extreme case where the corporate sector is taxed to finance subsidies and where firms are identical, have full knowledge of the method of financing subsidies, and take into account the response of its replicas, the subsidy has no effect. In less extreme cases a MES is always stimulating employment. Fourth, while the equilibrium effect of the subsidy is independent of risk attitudes, adjustment paths are generally affected.

Whenever the subsidy affects employment we find that an optimal subsidy exists and is equal to the maximum allowed by the finance constraints. Moreover, the same rate is optimal for all parties (government, firms, workers) involved. For some approximative empirical magnitudes for EEC countries the subsidy would bring these economies to target (full) employment within the short or medium run if for that time horizon the elasticity of labour demand exceeds 0,1.

To conclude we note some limitations of the analysis. Throughout the paper the heroic assumption of identical firms and homogeneous labour was maintained. If firms were diverse in their characteristics the results of the analysis would certainly be modified, but without specifying the difference in firm characteristics it is difficult to say more. One implication of diversity can however be noted from the analysis of cases 1 and 2 in Section 4. If firms are diverse (but still competitive) then case 2 is more likely to be applicable and the subsidy is more effective.

Finally, one would of course wish to abandon for some parts of the economy the hypotheses of competitive behaviour, introduce different skill categories for workers and model union behaviour. For example, it is not a surprise that an MES has the same employment effects in a closed as in an open economy as long as in both cases competitive behaviour prevails. One feature that may distinguish both sectors in the real world is a lower degree of competition in the closed economy (or sector) and in this context one would expect differential effects of an MES. However, introduction of such features would complicate the analysis considerably and our hopes of arriving at a firm theoretical characterisation are not strong enough to engage in this adventure. We still believe that our analysis captures the most essential aspects of the questions raised in this paper and that the results are meaningful approximations.

REFERENCES

K.J. Arrow and M. Kurz, Public Investment, The Rate of Return and Optimal Fiscal Policy, Baltimore : The Johns Hopkins Press, 1970.

J.N. Bhagwati and V.K. Ramaswami, "Domestic Distortions, Tariffs and the Theory of Optimum Subsidy," Journal of Political Economy, 71, February 1963, 44-50.

G.H. Borts, "Criteria for the Evaluation of Regional Development Programs," in W.Z. Hirsch, ed., Regional Accounts for Policy Decisions, Baltimore, 1966.

J.H. Drèze, "Salaires, emploi et durée du travail," Recherches Economiques de Louvain, 45, 1979, 17-34.

J.P. Gould, "Adjustment costs in the theory of investment of the firm," Review of Economic Studies, January 1968.

B. Holmlund, "Employment Subsidies and the Behaviour of the Firm," in G. Eliasson and J. Södersten, Business Taxation, Finance and Firm Behaviour, Stockholm : Almquist and Wicksell, 1978, 267-293.

N. Kaldor, "Wage Subsidies as a Remedy for Unemployment," Journal of Political Economy, 44, December 1936, 721-742.

J.R. Kesselman, S.H. Williamson and E.R. Berndt, "Tax Credits for Employment Rather than Investment," The American Economic Review, 67, June 1977, 339-349.

G.F. Kopits, "Comparison of Tax Depreciation Methods in Major Industrial Countries," International Monetary Fund, DM/80/60, 1980.

R.R.G. Layard and S.J. Nickell, "The Case for Subsidising Extra Jobs," The Economic Journal, 90, March 1980, 51-73.

R.E. Lucas, "Understanding Business Cycles", in Studies in Business-Cycle Theory, Cambridge, MIT Press, 1981.

M.I. Nadiri and S. Rosen, "Interrelated Factor Demand Functions", American Economic Review, 59, September 1969, 457-471.

OECD, Real Wages and Employment, DES/WP1/EM(82)2, March, 1982.

J. D. Sachs, "Wages, Profits and Macroeconomic Adjustment : A Comparative Study", Brookings Papers on Economic Activity, 2, 1979, 269-319.

A. Steinherr, "Why is unemployment so high in the EEC ?" (forthcoming, 1982).

Technical Appendix

A-1. Dynamic Analysis for Section 2

With the capital stock K held constant the optimal path for the optimisation problem posed in Section 2 is governed by the two differential equations

(1) $$\dot{N} = A - qN ,$$

(2) $$\dot{A} = \Phi^{-1}\{r + q)a_o A - \pi_N + (1 - \gamma)\frac{\pi_N a_o A}{\pi}(A - qN)\} ,$$

where

$$\Phi = a_o\{1 + (1 - \gamma)\frac{a_o A^2}{\pi}\} ,$$

and

$$\pi_N = pQ_N - (w - \sigma s).$$

The differential system (1) - (2) has been obtained from equations (6), (9) and (11) of the main text.

The Jacobian matrix of the differential system (1) - (2) is found to be

(3) $$J = \begin{bmatrix} -q & 1 \\ \theta & (r + q) \end{bmatrix}$$

where

$$\theta = -\{pQ_{NN} + (1 - \gamma)(r + q)qa_o^2A^2/\pi\}/\Phi .$$

The eigenvalues of the matrix are readily calculated and are

(4) $$\rho_1, \rho = \frac{r \pm \Delta}{2} ,$$

where

$$\Delta^2 = r^2 + 4\left[q(r + q)a_o + p|Q_{NN}|\right]/\Phi > r^2$$

and all quantities are evaluated at equilibrium.

It is readily found that

(5.a) $$\frac{d|\rho_2|}{d\gamma} = \frac{\{q(r + q)a_o + p|Q_{NN}|\}a_o^2q^2N^2}{\Phi^2\pi} > 0 ,$$

(5.b) $$\frac{d|\rho_2|}{d(\sigma s)} = -\frac{1}{\Delta\Phi}\{\frac{2(q(r + q)a_o + p|Q_{NN}|)q^2a_o^2(1 - \gamma)N}{\Phi\pi} + pQ_{NNN}\}\frac{\partial N}{\partial s} < 0 ,$$

and we have imposed $Q_{NNN} \geq o$ as would be the case with a Cobb-Douglas production function for example.

The eigenvectors of J corresponding to ρ_1, ρ_2 are respectively :

(6) $$V_1 = \begin{bmatrix} 1 \\ q + \rho_1 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 1 \\ q + \rho_2 \end{bmatrix},$$

so that the general solution of the differential system (1) - (2), in the neighbourhood of the equilibrium $(\bar{N}, \bar{A})$, may be written :

(7) $$\begin{bmatrix} N - \bar{N} \\ A - \bar{A} \end{bmatrix} = B_1V_1e^{\rho_1 t} + B_2V_2^{\rho_2 t} .$$

To pick out the stable arm of the local saddle-point we choose the as yet unspecified initial value A_o so as to equate B_1 to zero (thus eliminating the growing positive exponential term $e^{\rho_1 t}$); the known initial value N_o then determines B_2. As a result of these calculations we find that the time paths of N and A are given by

(8) $$N(t) = \bar{N} + (N_o - \bar{N})e^{\rho_2 t} ,$$

(9) $$A(t) = \bar{A} + (N_o - \bar{N})(q + \rho_2)e^{\rho_2 t}$$

Eliminating $(N_o - \bar{N})e^{\rho_2 t}$ between (8) and (9), we find that along the optimal path N and A are related via

(10a) $$N = (\bar{N} - \frac{\bar{A}}{q + \rho_2}) + \frac{A}{q + \rho_2} ,$$

from which it immediately follows that the slope of the optimal path in the (N, A) phase plane is

(10b) $$\frac{dN}{dA} = \frac{1}{q + \rho_2} < 0 .$$

The sign of (10b) follows observing that

$$\Delta^2 = r^2 + 4rq + 4q^2 + 4p|Q_{NN}|/a_o + r^2(1 - \gamma)a_o q^2 N^2/\pi > (r + 2q)^2$$

so that

$$q + \rho_2 < 0.$$

A-2. Impact Effects when Fiscal Deficits are Financed on External Capital Markets

For analytical tractability we consider the model of Section 3 with K held constant. So we are considering the optimisation problem

(11) $$\max_A \int_o^\infty e^{-rt} \pi^\gamma dt$$

s.t.

(12) $$\dot{N} = A - qN$$

(13) $$\dot{F} = \xi\sigma s(N - N_o) + w\alpha\xi(\frac{L}{n} - N) - fF$$

where $\xi = n$ for case 1 and $\xi = 1$ for case 2.

The first-order conditions are

(14) $\dot{\lambda}_1 = \gamma\pi^{\gamma-1} c_A ,$

(15) $\dot{\lambda}_1 - (r + q)\lambda_1 = - \{\gamma\pi^{\gamma-1} \pi_N + \lambda_2\xi(\sigma - w\alpha)\} ,$

(16) $\dot{\lambda}_2 - (r + f)\lambda_2 = \gamma\pi^{\gamma-1} (r + f)/n .$

Using (14) to turn (15) into a differential equation for A we have together with (12), (13) and (16) a system of four non-linear differential equations in A, F and λ_2. A local linear analysis of this system is only tractable for the special case $\gamma = 1$. In this case we find that the Jacobian matrix of the differential system governing the motion of N, A, F and λ_2 is

(17) $$J = \begin{bmatrix} -q & 1 & 0 & 0 \\ -pQ_{NN}/a_o & (r + q) & 0 & \dfrac{-\xi(\sigma s - w\alpha)}{a_o} \\ \xi(\sigma s - w\alpha) & 0 & -f & 0 \\ 0 & 0 & 0 & (r + f) \end{bmatrix}$$

This matrix has eigenvalues $(r + f)$, $-f$, ρ_1, ρ_2 and so the differential system for N, A, F and λ_2 exhibits local saddle point behaviour provided $f > 0$. The eigenvectors associated with the stable negative roots $-f$ and ρ_2 are respectively

(18) $$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ q + \rho_2 \\ \xi(\sigma s - w\alpha) / (f + \rho_2) \\ 0 \end{bmatrix}$$

With the same procedure as used to arrive at the solutions (8) and (9) for the model of Section 2 we find that

(19) $N(t) = \bar{N} + (N_o - \bar{N})e^{\rho_2 t}$

$$(20) \qquad A(t) = \bar{A} + (N_o - \bar{N})(q + \rho_2)e^{\rho_2 t}$$

$$(21) \qquad F(t) = \bar{F} + \{(F_o - \bar{F}) - \frac{(N_o - \bar{N})\ \xi(\sigma s - w\alpha)}{f + \rho_2}\}e^{-ft}$$

$$+ \frac{(N_o - \bar{N})\ \xi(\sigma s - w\alpha)}{(f + \rho_2)} e^{\rho_2 t} .$$

In the case $\xi = n$ we know from the main text that the subsidy s has no effect on $\bar{N}$, hence $\bar{N}$ remains at the initial level N_o and we see that in this case $N(t) = N_o$, $A(t) = qN_o$ and $F(t) = \bar{F}$.

For the second case $\xi = 1$, we know from the main text that $\bar{N}$ is affected by s, in particular that $\bar{N} > N$. We see by comparing (8), (9) with (19), (20) that the qualitative effect of a change in s on the dynamic parths of N and A are the same as in Section 2, at least for the special case $\gamma = 1$.

A-3. Optimal Subsidy Scheme

Referring to the optimisation problem (11), (12), (13) in A-2, s now also becomes a control variable. Imposing on the subsidy s the inequality constraint[1]

$$(22) \qquad \alpha w - s \geq 0$$

and appending this constraint to the Hamiltonian with the multiplier μe^{-rt} the necessary conditions become

$$(23) \qquad \lambda_1 = \gamma\pi^{\gamma-1}\ C_A ,$$

$$(24) \qquad \xi\lambda_2 = \frac{\mu}{\sigma(N - N_o)} - \gamma\pi^{\gamma-1} ,$$

$$(25) \qquad e^{-rt}\ \mu(\alpha w - s) = 0,\ e^{-rt}\ \mu \geq 0,\ \forall\ t ,$$

[1] Without a constraint on s the optimisation leads to contradictions for reasons which will be explained below. For convenience we reintroduce the constraint already used in Section 2.

together with the differential equations

(26) $\dot{\lambda}_1 - (r + q)\lambda_1 = -\gamma\pi^{\gamma-1}(pQ_N - w(1 - \alpha)) - \frac{\mu(\sigma s - \alpha w)}{\sigma(N - N_o)}$,

(27) $\dot{\lambda}_2 - (r + f)\lambda_2 = \gamma\pi^{\gamma-1}(r + f)/n$.

We find that the equilibrium values of A, N and s are given by

(28) $A = qN$

(29) $pQ_N = (r+q)C_A + w(1-\alpha) - \frac{\mu(\sigma s - \alpha w)}{\gamma\pi^{\gamma-1}\sigma_o(N - N_o)}$

and

(30) $(1 - \frac{\xi}{n}) = \frac{\mu}{\gamma\pi^{\gamma-1}\sigma_o(N - N_o)}$.

We are considering the case $\xi = 1$. It is clear from (30) that we must have $\mu > 0$ in order to avoid the contradiction $1 - 1/n = 0$. It then follows from (25) that the inequality constraint is saturated so that

(31) $s = \alpha w$

in equilibrium.

Combining (29),(30) and (31) we find that the equilibrium level of employment is given by

(32) $pQ_N = (r + q)C_A + w(1 - 1/n) - (1 - 1/n)\sigma_o\alpha w$.

Notice that if we had not imposed any contraint on s (so that $\mu = 0$ from the outset) then (30) would have become the contradiction $1 - 1/n = 0$. It is this contradiction which imposes on us mathematically the economically sensible constraint (22).

Finally in the case $\xi = n$ when, as we know from the main text, the subsidy scheme cannot affect employment, we see that (30) can only be satisfied if $\mu = 0$. The equilibrium level of s is then indeterminate, a further reflection of the fact that no optimal subsidy scheme exists in this case.

A-4. Employment subsidies in the Closed Economy

Real profits of the representative firm which are assumed to be consumed are given by

$$\pi = Q(K,N) - C(A,I) - wN - \alpha w(L - N) + \sigma s(N - N_o) - \frac{rV}{p} \quad . \tag{33}$$

The stock of capital is owned by workers and its nominal value at time t equals

$$V(t) = \int_{-\infty}^{t} p(v)\ I(v)dv \tag{34}$$

where, for simplicity, the rate of capital depreciation δ is set equal to zero. Real interest payments by the firm equal rV/p and are received by workers.

For a given time path p and s the representative firm seeks investment I and hiring A so as to maximise

$$\int_o^\infty e^{-rt}\ \pi^\gamma dt \tag{35}$$

subject to

$$\dot{N} = A - qN \tag{36}$$

$$\dot{K} = I \tag{37}$$

$$\dot{V} = pI \ . \tag{38}$$

Using λ_1, λ_2, λ_3 to denote respectively the shadow prices of labor, real capital, and the nominal value of capital the necessary conditions for the firm's problem may be written :

(39) $\lambda_1 = \gamma\pi^{\gamma-1}C_A$

(40) $\lambda_2 + \lambda_3 p = \gamma\pi^{\gamma-1}C_I$

(41) $\dot{\lambda}_1 - (r + q)\lambda_1 = - \gamma\pi^{\gamma-1}(Q_N - w(1-\alpha) + \sigma s)$

(42) $\dot{\lambda}_2 - r\lambda_2 = - \gamma\pi^{\gamma-1}Q_K$

(43) $\dot{\lambda}_3 - r\lambda_3 = \gamma\pi^{\gamma-1}r/p$.

The consumer's problem for a given path of p and s amounts to choosing real consumption E so as to maximise

(44) $\int_o^\infty e^{-rt}u(E)dt$

where u is a concave utility function. Since the work-leisure ratio is considered institutionally fixed the disutility of work or the utility from unemployment are constants and therefore absent from the function u.

From (41) - (43) of the main text and defining nominal wealth per worker as $W = nV/L$, the income constraint on workers can be written :

(45) $\dot{W} = p \{\frac{wN + \alpha w(L - N) - s(N - N_o)}{L}\} + rW - pE$

where $\dot{W}$ represents nominal savings.

Letting η denote the consumer's shadow price for wealth W, the necessary conditions are

(46) $\eta p = u'(E)$

(47) $\dot{\eta} - r\eta = - r\eta.$

Noting from (47) that $\dot{\eta} = 0$ these two equations combine to yield the differential equation determining the time path for real consumption E, which is

$$(48) \qquad \frac{\dot{p}}{p} = \frac{Eu''(E)}{u'(E)} \frac{\dot{E}}{E}$$

where $-Eu''(E)/u'(E)$ measures the relative risk aversion of consumers.

The representative firms' necessary conditions (39) - (43) imply a demand for investment schedule dependent on p and s, which can be aggregated over n firms. The workers-consumers' necessary condition (48) together with the income constraint (45) imply a supply of savings schedule also dependent on p and s, which can be aggregated over L workers-consumers . By equating the aggregate demand for investment to the aggregate supply of savings at every point of time we would obtain the differential equation determining the time path for p. We do not explicitly write down this differential equation since the dynamics of the model is rather complex. The equilibrium solution is however easily obtained and is given by :

$$(49) \qquad Q_N = (r + q)C_A + w(1 - \alpha) - \sigma_o s$$

$$(50) \qquad Q_K = r(1 + C_I)$$

$$(51) \qquad LE = wN + \alpha w(L - N) - \sigma_o s(N - N_o) + rnV/p \ .$$

We observe that (49) and (50) are independent of prices and identical to (15) and (16) of Section 2 so that the equilibrium effects of the employment subsidy on employment and the capital stock are the same. Condition (46) together with (51) determine the equilibrium level of consumption by workers and the price level.