

ECONOMIC PAPERS

COMMISSION OF THE EUROPEAN COMMUNITIES • DIRECTORATE-GENERAL FOR ECONOMIC AND FINANCIAL AFFAIRS

N° 37

June 1985

SCHEMAS FOR THE CONSTRUCTION OF AN
"AUXILIARY ECONOMETRIC MODEL"
FOR THE SOCIAL SECURITY SYSTEM

A. COPPINI*
assisted by
G. LAINA*

Internal Document



"Economic Papers" are written by the Staff of the Directorate-General for Economic and Financial Affairs, or by experts working in association with them. The "Papers" are intended to increase awareness of the technical work being done by the staff and to seek comments and suggestions for further analyses. They may not be quoted without authorisation. Views expressed represent exclusively the positions of the author and do not necessarily correspond with those of the Commission of the European Communities. Comments and enquiries should be addressed to

The Directorate-General for Economic and Financial Affairs,
Commission of the European Communities,
200, rue de la Loi
1049 Brussels, Belgium

ECONOMIC PAPERS

N° 37

June 1985

SCHEMAS FOR THE CONSTRUCTION OF AN
"AUXILIARY ECONOMETRIC MODEL"
FOR THE SOCIAL SECURITY SYSTEM

A. COPPINI*
assisted by
G. LAINA*

Internal Document

A. COPPINI is professor of actuarial techniques for social insurance at La Sapienza University, Rome. He is a former president of the Italian sickness insurance institute (INAM) and member of the European Communities Economic and Social Committee. His published works include a treatise on social insurance techniques and an essay on redistribution, as well as a number of notes and articles. For the past few years, he has concentrated his studies on the economic aspects of social security.

G. LAINA is an official with the Directorate-General for Economic and Financial Affairs, Commission of the European Communities. He helped to devise and set up EUROLINK, which has contributed to several research projects into social security.

Summary

This paper describes the structure of a sub-model of the social security system which can be used by itself or linked to a general model of the economy. Construction of the sub-model is based on a specification, containing some novel features, of the various social security benefits and contributions using actuarial and econometric methods. In this paper the emphasis is placed on benefits, the research on which is the most original. The paper concludes with the testing of one of the algorithms proposed within the framework of a large econometric model.

In the final analysis the sub-model aims at measuring the economic effects of the social security system on the key economic aggregates, especially in the event of legislative changes.

<u>TABLE OF CONTENTS</u>		<u>Page</u>
1.	INTRODUCTION	1
1.1.	<u>General remarks</u>	1
1.2	<u>Objectives and characteristics of the "auxiliary model"</u>	4
1.3	<u>Some words of advice</u>	6
2.	PENSION COSTS	6
2.1	<u>General remarks</u>	6
2.2	<u>Determination of the pensioned population</u>	7
2.3	<u>Determination of the average benefit</u>	14
2.4	<u>Determination of total costs</u>	
3.	FAMILY ALLOWANCE COSTS	16
3.1	<u>General remarks</u>	16
3.2	<u>Determination of the average allowance</u>	17
3.3	<u>Determination of total costs</u>	18
3.4	<u>Additional comments</u>	18
4.	TEMPORARY DISABILITY COSTS	19
4.1	<u>Characteristics of benefits</u>	19
4.2	<u>Determination of the average annual cost per worker</u>	20
4.3	<u>Forecasts of total costs</u>	21
4.4	<u>Additional comments</u>	21
5.	UNEMPLOYMENT COSTS	22
5.1	<u>Forms of protection</u>	22
5.2	<u>Determination of the average annual number of unemployed persons in receipt of benefit</u>	22
5.3	<u>Determination of the costs of unemployment benefit</u>	25
5.4	<u>Underemployment costs</u>	25
6.	HEALTH CARE COSTS	26
6.1	<u>General remarks</u>	26
6.2	<u>Determination of average costs</u>	27
6.3	<u>Forecast of total costs</u>	29
7.	SOCIAL SECURITY CONTRIBUTIONS; OTHER REVENUE; OTHER EXPENDITURE	30
7.1	<u>Contributions</u>	30
7.2	<u>Other revenue; other expenditure</u>	30

(suite of table of contents)

8.	REDISTRIBUTION EFFECT	30
8.1	<u>General remarks</u>	30
8.2	<u>Basic data and assumptions</u>	32
8.3	<u>Determining the redistribution effect</u>	34
8.4	<u>Additional comments</u>	
9.	APPLICATION OF THE SUGGESTED PROCEDURE: AN EXAMPLE	38
9.1	<u>Conditions of the experiment</u>	38
9.2	<u>Findings</u>	38
10.	SOME CONCLUSIONS OF THE PROBLEM OF SPECIFICATION	41
	APPENDIX (technical explanations, abbreviations and references)	43-50
	FOOTNOTES	51-52

SCHEMAS FOR THE CONSTRUCTION OF AN
"AUXILIARY ECONOMETRIC MODEL" FOR THE SOCIAL SECURITY SYSTEM

1. INTRODUCTION

1.1 General remarks

1.1.1 Present difficulties of the social security funds and objective of this paper

Social security funds are coming in for much criticism at the moment, not only because they are experiencing financing problems, but also because of the macroeconomic effects they engender.

It is argued that the European economies, which were the first to apply principles of solidarity and social insurance, are now suffering from too much security in relation to market risks, and that this excessive security is one of the main reasons for mounting unemployment. A comparison of economic developments from 1970 to 1983 in Europe and the United States (where solidarity is not so generalized) shows that, while Europe has lost 1 million jobs, 17 million new jobs have been created in the United States.

The attempt to provide too high a degree of protection from market risks thus seems, perversely, rather to increase exposure to risk, at least if the risk of unemployment is regarded as a danger for society and for the individual.

There is no doubt that this argument has some basis in fact; but it is necessary to identify the situations that, by effectively eliminating economic risk, damage market efficiency.

Clearly, market suffers when unproductive firms are kept in business thanks to transfers paid from taxpayers' money without economic justification. Clearly also, productive firms require a certain amount of freedom to hire and fire, and to determine working hours, if they are to face up to competition and survive market pressures.

There is no particular reason to suppose that private individuals attempting to protect themselves against risks or to build up a capital sum for old age will introduce rigidity into the market for goods and services. Moreover, social insurance can be provided either by the state or by private undertakings, the only difference being a financial advantage to the individual if the service is provided by the state, since the state would not require remuneration in the form of profit. Of course, the state may fail to ensure balance between contributions and benefits, thus giving rise to adverse economic effects.

Surprising as it may seem, no measurements have been made so far to determine whether the various sectors of social security are in equilibrium overall, and if not, what would be the best means of restoring equilibrium. The requisite statistical information is lacking, as are precise studies of the behaviour of the different areas of social security. It is up to national government departments to supply the first lack; this study is an attempt to supply the second, by formulating, for the first time, a precise quantitative framework that can be adapted to allow for national variants and for each sector of activity.

1.1.2 The origin of the research

When I was preparing a report for the Meeting of Experts on Social Security Financing held by the International Labour Office in Geneva at the end of November 1982¹, I made two findings :

- (i) Analyses which are merely qualitative are in my opinion quite inadequate for the purpose of arriving at any valid decision, partly because there is no one single "truth" but many incidental and piecemeal truths depending on each country's circumstances and the period covered by the analysis itself;
- (ii) The costs of social security systems are characterized by a rigidity not encountered under other public finance headings; but in the econometric models of the larger countries and the description of such systems is summary and partly exogenous.

¹ See footnotes page 51.

I have found support especially for the second finding, in the work of two Belgian authors², who have reached similar conclusions and have built a "module" for the social security system, called SOZEC, which functions in the same way as the general MARIBEL model designed for study of the Belgian economy.

These considerations formed the basis of my intention to study in depth the problems of building an "auxiliary model"; this paper is a preliminary methodological contribution, which I hope to follow up shortly with an application for Italy.

1.1.3 Further explanations

In order to avoid misunderstandings I should like to make two further points.

Firstly, I am aware that "econometric models", like any other instrument of quantitative analysis, are far from perfect. Models are simplified representations of reality which - at least very largely - extrapolate in deterministic form the implications of an economic structure inferred from estimates made on past data. Furthermore, a satisfactory description of the phenomena concerning the social security system is gravely hampered by the paucity of and gaps in the relevant statistics.

Nevertheless, the results obtained by using models are probably more reliable than those obtained by other investigative methods, and there are prospects of increasing this reliability by improving the method (this paper is an attempt to do so).

Secondly, it may be useful to state that by "social security" I mean the forms of welfare protection which give rise to specific rights for the citizens (i.e. excluding cases where assistance may be optional) and which comprise the following branches: old-age, invalidity and survivors' pensions, temporary disability allowances, benefits for unemployment or short-time working, health care benefits, family allowances and accident benefits.

1.2 Objectives and characteristics of the "auxiliary model"

1.2.1 General and special objectives

The abbreviations MASS will hereafter be used to refer to the "auxiliary social security model" and the word "Model" to refer to the "general model" to which the auxiliary model may possibly be linked.

There are two objectives which justify constructing the MASS:

A. Analytical representation of the entire social security system in operation in a specific country (contributions, other items of income and expenditure, benefits, balances between income and expenditure and capital account balances) for the purpose of making:

- (i) autonomous projections based on exogenous hypotheses concerning changes in employment, incomes and prices;
- (ii) the same forecasts, taking the above-mentioned hypotheses from the Model and providing it with the relevant data.

B. Certain special analyses, to be made in particular when important reforms are pending; these relate to:

- (i) the social security system's income redistribution effects;
- (ii) the effects on costs, and potentially on prices, stemming from changes in the system of financing;
- (iii) the effects which variations in financing and in benefits have on consumption.

The objectives listed under A will be called "general" objectives, and those under B "special" objectives.

Each of these objectives will be defined in greater detail in the chapters which follow; here I wish merely to indicate the advantage of this method, namely that:

- (i) it permits an autonomous representation of the social security system with the details essential for an appraisal of its macroeconomic action, independently of the link with the Model;

- (ii) it provides the Model with more accurate projections by stripping it of equations and algorithms which are unwieldy compared with those which must be devoted to other sectors of the economy;
- (iii) it permits the study of legislative changes and of their implications even for those of the Model's variables which do not directly relate to the social security system.

1.2.2 Characteristics of the treatment

The main features of the procedure described below may be summarized as follows.

Attainment of the general objectives (projection of income, expenditure and balances) is simple and, in general, consistent with the available data; in addition to distinguishing between branches, account is also taken of the distribution by groups of schemes and economic sectors; this is because of the stated intention to perform more extensive tasks than simply providing the Model with assistance, and because we wish to avoid unduly rough approximations.

The procedures suggested for the special objectives (redistribution, effects on costs and on consumption) require more complex computations and statistics that are not in current use; however, once acquired, these statistics need only infrequently to be updated, and this practice has no adverse effect on the reliability of the results.

Lastly - and this is perhaps the most important point - for the general objectives the regression method has been replaced by procedures largely drawn from actuarial techniques. This makes it possible to take into account:

- (i) all the consequences of the rules in force (even where the consequences are postponed) or of the rules which it is desired to introduce;
- (ii) exogenous ad hoc forecasts (demographic projections, frequency of benefits, average costs, etc.);
- (iii) the starting situation, i.e. the structure of the insured and pensioned population at the start of the computation and the population's economic and insurance profile.

1.3 Some words of advice

As the title of this paper indicates, the treatment has been kept general and relatively succinct.

The treatment is general because the various schemas do not refer to a specific country or to specific rules. Further work will be needed to adapt the schemas described to an actual case. Furthermore, a particular care is required at the time of application in order to achieve a complete agreement between the magnitudes which are the subjects of the projection, their classification and the schemes of national book-keeping. The justification of particular developments had been reduced to a minimum in order to clarify the picture.

Certain cross-references to other pieces of work of mine may be useful.

The reader is advised to start by acquainting himself with the notation, which for ease of consultation is shown in an appendix at the end of the paper. In the individual chapters, no definition is given of conventions and symbols used in their general form. However, some polyvalent symbols specific to a chapter are defined when they are introduced.

2. PENSION COSTS

2.1 General remarks

Benefits paid in the event of disability and death due to non-occupational causes (for occupational causes, see chapter 8) and benefits for old age consist substantially of pensions, as a rule related to the number of years of contribution served.

The purpose of this section is to determine for some years, starting from a certain initial period:

- (i) the number of pensions in payment in the different categories;
- (ii) the average amounts involved.

The suggested procedure is a simplified version of the traditional projection methods used in social insurance technique and refers to a single scheme or to a group of schemes. In practice it will be useful to consider the general scheme separately and then to deal with the others in groups based on similarities in their rules. For Italy, for example, the following might be considered : the general scheme; the group of schemes for the self-employed (farmers who cultivate their own land, tradesmen and craftsmen); the group of schemes for public employees; two groups for all the other special schemes for wage and salary earners and for the professions; and the group of complementary schemes.

It is assumed below, in accordance with Italian law, that invalidity pensions continue to be paid beyond the qualifying age for the old age pension. It would not be difficult, however, to adapt the schema to the case of invalidity pensions ceasing to be paid when this age is reached and being replaced by old-age pensions.

2.2 Determination of the pensioned population

2.2.1 Overall data assumed to be available

With reference to any one scheme, it is assumed that at time 0 the following overall numbers of insured persons and pensioners in the three categories (invalids, old-age pensioners and survivors) are available : n_{0a} , n_{0i} , n_{0v} , n_{0f} .

The data in question will be assembled below into a row vector which we shall indicate as $\hat{n}_{p,0}$.

Let us also assume that for any $t = 1, 2, \dots$ subsequent years, the number of persons insured under the scheme in question n_{1a} , n_{2a}, \dots is given a priori.

In the absence of other information, especially for the national schemes, these numbers could be obtained from the expression

(2.1) $n_{t^a} = n_{0^a} \times (1+\delta_0) \times (1+\delta_{t-1})$ where the rates of increase (or decrease) in the working population are given exogenously or provided by the Model. These numbers will therefore, be substituted from time to time in the vectors $\hat{n}_{p,t}$ for those obtained by the computation described in subsection 2.2.3.

2.2.2 Matrix of elimination or change of status probabilities

We can write the following matrix

$$(2.2) \quad \hat{Q}_p = \begin{vmatrix} 1-q_a & q_a/i & q_a/v & q_a/f \\ 0 & 1-q_i & 0 & q_i/f \\ 0 & 0 & 1-q_v & q_v/f \\ 0 & 0 & 0 & 1-q_f \end{vmatrix}$$

For computation of the values contained in the matrix \hat{Q}_p we refer the reader to subroutine 2.1 (see subsection 2.2.4).

2.2.3 Computation of the pensioned population

The number of pensioners in the different groups (invalids, old age pensioners and survivors) is computed by means of the recurring formula

$$(2.3) \quad \hat{n}_{p,t} = \hat{n}_{p,t-1} \times \hat{Q}_p$$

We see that, by making the appropriate adjustments and constructing suitable \hat{Q}_p matrices, separate projections can be obtained of pensions in payment at time 0 and for those paid in subsequent years.

2.2.4 Subroutine 2.1 for computation of the probabilities contained in \hat{Q}_p

In order to compute the individual probabilities contained in \hat{Q}_p , availability of the following age and sex distributions is necessary:

$$g_{x^a}, g_{x^i}, g_{x^v}, g_{x^f}^3$$

For the general scheme, the distribution g_{x^a} may be replaced by the distribution for the working population.

The other distributions do not necessarily have to be those at time 0 but may refer to earlier periods or to similar experiences.

In addition it will be sufficient to have data grouped in five-year classes.

It is assumed further that the probabilities

$$1 - q_{x^a}, q_{x^a}/i, q_{x^a}/v$$

and the probabilities

$$q_{x^v}, q_{x^i} \quad 4$$

are available.

The probabilities q_{x^f} can be constructed by means of a simulation procedure⁵.

The probabilities q_{x^i}/f , q_{x^v}/f and the probabilities q_{x^a}/f are computed on the basis of the death probabilities of the groups concerned, applying the probabilities of leaving survivors.

With the above elements, all the probabilities contained in \hat{Q}_p are determined by means of appropriate averages. Thus for example

$$(2.4) \quad q_{x^a}/i = E(q_{x^a}/i, q_{x^a}).$$

A further observation must be made in order to construct the probabilities contained in \hat{Q}_p .

These probabilities may depend on t , and hence the matrix \hat{Q}_p may be replaced by matrices $\hat{Q}_{p,t}$, if

- (a) the distribution q_{x^a} , q_{x^i} , q_{x^v} , q_{x^f} vary over time;
- (b) the different sex and age probabilities can vary very rapidly over time owing to natural causes (e.g. reduction or increase in invalidity);
- (c) rules are changed in such a way as to involve changes in the aforementioned probabilities.

The actual distribution under (a) and the probabilities under (b) no doubt vary over time, but in the period covered by medium-term forecasts (4 to 5 years) they can for simplicity's sake be assumed to be constant.

However, if changes are made in the rules - e.g. if the minimum qualifying age for the old-age pension is lowered or raised - the corresponding probabilities must be adjusted according to criteria to be studied case by case.

2.2.5 Further explanations and comparisons

The purpose of the foregoing calculations is to improve the accuracy of forecasts of the population in receipt of pensions. As a comparison of the results with those of a conventional econometric model show, the findings of the iterative process are more consistent with observable facts than the results that can be obtained from the specifications usually used for econometric models.

We will first examine a case of a "traditional" specification based on the method of regression and for this purpose we shall refer to the French model METRIC.

In this model for old-age and survivors there is the following linear equation

$$(I) \quad \boxed{\begin{array}{l} \text{old-age and} \\ \text{survivor} \\ \text{benefits} \end{array}} = a + b \times \boxed{\begin{array}{l} \text{deferred wages} \\ \times \\ \text{population of} \\ \text{widows } > 60 \\ \text{and men } > 65 \end{array}} + c \times \boxed{\begin{array}{l} \text{minimum} \\ \text{pension} \\ \times \\ \text{popula-} \\ \text{tion } > 65 \end{array}}$$

The parameters a, b and c are in fact estimated by regression and the "deferred" wages consist of an appropriate average of the wages of the four quarters preceding the date of calculation.

In this equation one does not show in any way the pensions in payment nor their average amount : we in fact employ quantities which

only indirectly - and relatively imperfectly - represent the trend in costs. On the other hand the procedure, once the parameters have been estimated - which it is important to note should be changed if the provisions of the scheme are revised - is particularly simple and therefore suitable for inclusion among the very large number of other equations of the general model.

Under the auxiliary models the first example is drawn from SOZEC.

In this model the forecast of benefits makes use of a general design of the "mechanical" type, a procedure which links together the following :

- the elements that derive from the assumed provisions in force;
- the factors that result from exogenous valuations carried out ad hoc (demographic forecasts, benefit frequencies, average costs at constant prices, etc.);
- the economic data largely drawn from the general model (prices, wages, employment, etc.).

Leaving aside the minor and optional benefits the diagram is as follows :

$$(II) \quad \boxed{\text{benefits at time } t} = \boxed{\text{benefits at time } t - 1} \times \boxed{\text{index of quantity}} \times \boxed{\text{consumer price index}}$$

We should consider for a moment the index of quantity which is the key element in the projection and which can also be defined as the index of benefits at constant prices.

In the specific case of pensions this index derives from a product and thus it is itself the expression of a procedure of definition:

$$(III) \quad \boxed{\text{index of quantity}} = \boxed{\text{index of number of beneficiaries}} \times \boxed{\text{index of the average benefit at constant prices}}$$

We should add that this index is in turn an appropriate average of indices constructed separately for individual ages (or age groups) of the beneficiaries.

Thus, in SOZEC, there are included as a determining element the distributions, in this case those covering the pension beneficiaries.

Somewhat more complicated and closer to actuarial method is the procedure proposed in this paper.

In these designs we also suppose that the calculation will be made separately by schemes or groups of schemes.

For each scheme the valuation is based on a first "defining" equation which is, however, expressed in vector form :

$$(IV) \quad \boxed{\begin{array}{c} \text{annual} \\ \text{benefits} \end{array}} = \boxed{\begin{array}{c} \text{vector of number} \\ \text{of beneficiaries} \end{array}} \times \boxed{\begin{array}{c} \text{vector of average} \\ \text{benefits} \end{array}}$$

The vectors are in three columns, which in fact serve to keep separate the pensions for the three basic types (invalidity, old-age and survivors).

The vector of average benefits is constructed by ad hoc procedures which vary from one scheme to another, and on which lack of space prevents us from dwelling, so we confine ourselves to the fact that they take into account :

- data from experience at the time of calculation;
- the adjustment provisions that are currently in force or assumed;
- the indices of prices and/or of wages drawn from the general model.

We do, on the other hand, look in some detail at how the vector of beneficiaries is constructed.

Let us suppose that at time zero we have a 4-column vector which shows both the number of the existing pensions and the number of insured

persons, and we further construct a matrix with 4 rows and 4 columns which summarises the probabilities, independent of age of withdrawal from active status (for the various reasons : death, invalidity, old-age, etc.), those of elimination from pensions of the various groups, those remaining in active status and in pensionable status (in each case separately for the 3 groups).

The vector of active persons and pensioners at time 1 will then be provided by the product ⁶ of :

$$(V) \begin{array}{|l|} \hline \text{vector of number} \\ \text{of active persons} \\ \text{and beneficiaries} \\ \text{at time 1} \\ \hline \end{array} = \begin{array}{|l|} \hline \text{vector of active} \\ \text{persons and} \\ \text{beneficiaries} \\ \text{at time 0} \\ \hline \end{array} \times \begin{array}{|l|} \hline \text{matrix of the} \\ \text{probabilities of} \\ \text{elimination and} \\ \text{of remaining} \\ \hline \end{array}$$

Basically, the vector of active persons and beneficiaries is written as follows:

$$n = [\text{active persons, invalids, retired persons, survivors}]$$

The matrix of elimination and remaining contains the following frequencies the sum of which is normally less than one:

active population:	frequency of remaining active	1-qa
"	" : frequency of taking invalidity	qa/i
"	" : frequency of taking retirement	qa/v
"	" : frequency of new entitlement to survivors pension	<u>qa/f</u>

invalid population:	frequency of remaining such	1-qi
"	" frequency of taking retirement	0 ⁷
"	" frequency of new entitlement to survivors pension	<u>qi/f</u>

retired population: frequency of remaining such ... etc.

Some observations are still necessary in order to understand the procedure.

In the first place it is obvious that if (IV) is related to year 1, the first vector of the second member is obtained from the first member of (V), simply by eliminating the number of active persons.

Secondly, it is also evident that the procedure is of a recurring nature, subject only to introducing in the various vectors which one obtains by (5) the number of active persons arrived at by exogenous means.

Finally - and here one again sees the use of distributions - one has to point out that the probabilities of the matrix are obtained as averages between the corresponding probabilities by age (or age group), obtained by appropriate distributions of the active persons and the pensioners at time 0. The distributions form part of the auxiliary model and can be updated as and when new experience and information are available.

2.3 Determination of the average benefit

2.3.1 Static situation

The determination of average annual benefits

$$\bar{P}_t i, \bar{P}_t v, \bar{P}_t f$$

in a static situation cannot be based on general rules because each scheme has specific rules governing the relationship of the pension value to wages (or contributions).

Certain comments can, however, be made :

- (i) It can be assumed that the following values will be available :
 $\bar{P}_0 i, \bar{P}_0 v, \bar{P}_0 f;$
- (ii) If pensions do not depend on the number of years of contribution served, or if the system has been in operation for a long time (50 years or more) and the average number of years of contribution of new pensioners will presumably not increase, the average values recorded at time 0 can be adopted;

(iii) Otherwise, by using the computation based on the recurring formula (2.3), each year's new invalidity, old-age and survivors' pension can, as has been stated, be kept separate, so that, by means of ad hoc valuations, the initial benefits \bar{p}_{0i} , \bar{p}_{0v} , \bar{p}_{0f} can be used to establish those of subsequent years by increasing the initial benefits accordingly⁸.

2.3.2 Corrections to take account of the adjustment clauses

After adjustment, pensions hardly ever conform exactly to changes in wages or incomes. Sometimes adjustment is limited to a single factor (e.g. the value of money), or is postponed, or affects only part of the pension, etc. For each pension category it is therefore essential to compute the coefficients β_t (generally smaller than 1) as a function of the rates ρ of increase in wages.

2.4 Determination of total costs

2.4.1 Static situation

Total costs in a static situation can be determined immediately with the elements given above.

At the end of year t we have

$$(2.7) \quad P_t P = \hat{n}_{p,t} \times \hat{p}_{p,t}$$

where

$$(2.8) \quad \hat{p}_{p,t} = \left\| \begin{array}{l} 0; \bar{p}_{ti}; \bar{p}_{iv}; \bar{p}_{if} \end{array} \right\|$$

2.4.2 Dynamic situation

For costs in a dynamic situation, assuming that the rates of increase in wages have been assigned, at the end of year t we shall have

$$(2.9) \quad \widetilde{P}_t P = P_t P \times (1+\rho_0) \times \dots \times (1+\rho_{t-1}) \times \bar{\beta}_t.$$

3. FAMILY ALLOWANCE COSTS

3.1 General remarks

3.1.1 Benefits considered

In all countries the most substantial benefit payable to families is the family allowance; this will be the sole subject of discussion here.

The procedure described below relates to a specific scheme and must therefore be repeated for the different schemes or groups of schemes to be considered.

It is recommended that several systems should be grouped together (e.g. wage and salary earners, public employees, self-employed workers); if the allowance are the same for all schemes, a single computation can be made.

3.1.2 Types of benefits

Family allowances are paid on one of the following principles :

- (i) a fixed amount which is the same for all family situations;
- (ii) a fixed amount which varies with the family situation;
- (iii) an amount which is the same for all family situation but varies with the wage bracket;
- (iv) an amount proportionate to the wage or salary.

More than one of the above principles may, of course, be applied simultaneously.

The procedure described is based on the hypothesis that the allowances depend on family composition and wage category.

3.2 Determination of the average allowance

3.2.1 Static situation

Let us suppose that the double distribution $g_{f,wa}$ is available for workers who are heads of households according to the composition of the dependent family⁹ and the wage category.

In practice it will be sufficient to have two separate unitary distributions g_{fa} and g_{wa} for family composition and wage category respectively, since it is possible by close approximation to obtain some value for $g_{f,wa}$ as the product of the corresponding values of g_{fa} and g_{wa} . In every case it is helpful to use separate distributions for the sex of the head of household.

For the distribution g_{fa} , the appropriate aggregations can also be introduced in relation to the rules governing the allowances (e.g. if there are no separate amounts for the spouse and the children, one distribution for a number of components will be sufficient).

Now if P_{fwb} ¹⁰ represents the unit benefits (per year and per family component) in accordance with family composition and wage category, the values in question may be assembled into a matrix which we shall indicate as \hat{P}_b ; by analogy the values of the distribution $g_{f,wa}$ give rise to a matrix, with the same number of rows and columns, which we shall indicate as \hat{G}_a . For the average benefit we shall therefore have

$$(3.1) \quad \bar{p}_{ob} = E(\hat{P}_b; \hat{G}_a) = \frac{\hat{e} \times \hat{P}_b \times \hat{G}_a \times \hat{e}'}{\hat{e} \times \hat{G}_a \times \hat{e}'}$$

3.2.2 Dynamic situation

Over time the following may vary :

- (i) family composition;
- (ii) distribution by wage category;
- (iii) the amount of the allowance if they are indexed in any way.

For the first element the hypothesis of invariance can be formulated without causing appreciable errors.

For the wage-category distribution it is sufficient, as a first approximation, to vary the category limits in keeping with the average increase in wages and salaries. In more complex wage-adjustment conditions (e.g. where increases are in fixed amounts for all categories), the problem must be studied case by case.

Appropriate modifications must also be introduced for allowances indexed according to the rules that govern them.

3.3 Determination of total costs

3.3.1 Static situation

Total costs are computed directly when the number n_{0a} is known.

$$(3.2) \quad P_{tB} = n_{0a} \cdot (1+\delta_0) \cdot \dots \cdot (1+\delta_{t-1}) \cdot \bar{p}_{0b}$$

where δ indicates the rate of increase in heads of households covered by the scheme under consideration. This rate may be obtained from the Model which establishes the proportion of workers in employment, except in the case of special-category schemes (e.g. for public employees), for which ad hoc forecasts will be necessary.

3.3.2 Dynamic situation

The computation is similar in a dynamic situation. We have :

$$(3.3) \quad \tilde{P}_{tB} = n_{0a} \cdot (1+\delta_0) \cdot \dots \cdot (1+\delta_{t-1}) \cdot \bar{p}_{tb}$$

3.4 Additional comments

Family allowances - when not distributed to the entire population - are usually paid to :

- (i) the temporarily disabled;
- (ii) the unemployed;
- (iii) pensioners.

In the first two cases the average benefits computed in section 3.2 may also hold good and be aggregated with the principal benefit : where different rules apply, the distribution $g_{f,w}$ (or g_f alone) can be used in each case. For pensioners, in order to speed up the computation, it is preferable for the allowances to be included directly in the computation of average pensions.

4. TEMPORARY DISABILITY COSTS

4.1 Characteristics of benefits

The allowances which replace the wage or salary in the event of temporary disability due to sickness are usually paid to wage and salary earners who have no stable employment.

The benefit almost always equals a percentage of the wage or salary.

The subsidiary conditions, however, vary. These are :

- (i) a waiting period of one or more initial days, which may be fixed or proportionate; some rules do not provide for a waiting period;
- (ii) a maximum period of payment, which may be related to each case of sickness, or to the number of days of sickness registered in each year, etc.;
- (iii) possible differences in the level of benefit depending on the period over which it is paid (e.g. a smaller percentage for long illnesses).

Here let us consider the case of an allowance which is fixed at a single percentage for the entire period of payment, to which no waiting period applies, and which is paid for the maximum period for the type of illness concerned (in accordance with the rules in force in Italy).

Similar procedures may be used to extend the computation to other situations.

4.2 Determination of the average annual cost per worker

4.2.1 Static situation

The basis of calculation is represented by what is broadly termed "morbidity coefficient" ζ_x , i.e. the number of days for which sickness benefit is paid during the year for an insured person aged x .

For our purposes we need an age distribution g_{xa} and must determine the average coefficient.

$$(4.1) \quad \bar{\zeta} = E(\zeta_x; g_{xa}).$$

Given the limited variation in the age distribution within the brief forecasting period, g_{xa} may remain unchanged; it is also possible to use an average coefficient $\bar{\zeta}$ directly recorded. For this benefit it is, however, important to keep the two sexes separate.

If γ is the corresponding percentage and \bar{w}_0 the average wage of all the workers concerned, for the average annual benefit we shall have

$$(4.2) \quad \bar{p}_0^m = \frac{\gamma \cdot \bar{w}_0 \cdot \bar{\zeta}}{365}$$

4.2.2 Dynamic situation

The average benefit in a dynamic situation must first take account of the wage variations which occur in certain years. These variations are supplied by the Model or may be derived from independent assumptions.

If $\rho_0, \rho_1 \dots$ are the corresponding rates, we shall have :

$$(4.3) \quad \bar{p}_t^m = \bar{p}_0^m \cdot (1+\rho_0) \dots \cdot (1+\rho_{t-1}).$$

The variations in the coefficients ζ_x and hence in the average coefficient $\bar{\zeta}$ may be another dynamic factor. No general rules can be laid down on this point, except that the preceding years' trend should be

taken into account (if the conditions for benefit are unchanged). If these findings indicate an appreciable variation, this variation can be extrapolated for the years to which the computations refer; a coefficient τ variable over time will be adopted accordingly.

4.3 Forecasts of total costs

4.3.1 Static situation

If the number n_{0a} of workers insured at time 0 is known, we immediately obtain for total costs

$$(4.4) \quad P_t M = n_{0a} (1 + \delta_0) x \dots x (1 + \delta_{t-1}) x \bar{p}_{0m}$$

where the δ symbols indicate the rates of variation in the insured population provided by the Model or determined ad hoc.

4.3.2 Dynamic situation

In a dynamic situation we have, by analogy with (4.4)

$$(4.5) \quad \tilde{P}_t M = n_{0a} x (1 + \delta_0) x \dots x (1 + \delta_{t-1}) \bar{p}_{tm}.$$

4.4 Additional comments

4.4.1 Maternity benefits

These benefits, which consist of the maternity allowance paid before and after confinement, may be treated in exactly the same way, by applying to the female population appropriate coefficients arrived at as the product of the frequency of births (per age) times and the number of days of leave granted.

4.4.2 Changes in the conditions for benefits

If changes are made in the waiting period and the limit of duration, the coefficients ζ_x , which depend on the rules in force in the period under observation, will clearly have to be modified. Reduction factors with very simple formulae can be used for this purpose; for these formulae we refer the reader to the ext mentioned in the footnote¹¹.

5. UNEMPLOYMENT COSTS

5.1 Forms of protection

Protection in the event of unemployment and underemployment varies widely in organization from country to country. Certain typical forms of assistance can be indicated; these are :

- a) the daily allowance for wage and salary earners who have lost their employment; this allowance is generally a fixed amount, and it is paid within the limits set by the waiting period and the maximum period of payment;
- b) assistance for unemployment persons who are not (or are no longer) entitled to the benefit described under (a) and for those awaiting their first employment; these kinds of assistance take the form of grants and are sometimes optional and temporary;
- c) supplements to the earnings of wage and salary earners for firms which are compelled to introduce short-time working or to exclude groups of workers from the productive process for some months; the benefit consists of a percentage of the wage lost for a maximum period stipulated in the rules.

Since we cannot examine all the possibilities, we shall here consider explicitly the benefits described under (a), which are independent of the wage and are contained within the limits set by an absolute waiting period and a maximum period of payment. We shall then provide some indications regarding the benefit described under (c).

As usual we shall refer to a specific scheme, even though in this case there is often one single scheme.

5.2 Determination of the average annual number of unemployed persons in receipt of benefit

5.2.1 Characteristics of the problem

The primary basis of calculation for determining the benefits under consideration must be the number of unemployed per year or the rate supplied by the Model (or independently assumed). This means that evaluation takes place directly in a dynamic situation.

However, the problem is not a simple one because it is necessary to subtract from the above average number :

- (i) the unemployed who are not entitled to benefit (e.g. self-employed workers or persons awaiting their first employment);
- (ii) those who, although insured, are no longer entitled to benefit because the maximum period of payment has expired, and those who are awaiting the end of the fixed waiting period.

5.2.2 Computation procedure

Let us suppose first of all that we know the numbers n_{0a} of workers covered by unemployment insurance at time 0. Let us compute the numbers of persons insured at time t by the formula :

$$(5.1) \quad n_{ta} = n_{0a} \cdot (1 + \delta_0) \cdot \dots \cdot (1 + \delta_{t-1})$$

where the rates δ are the rates of variation in numbers of persons insured provided by the Model (in the case of a general scheme) or estimated ad hoc.

Then let :

λ_t be the unemployment rate supplied by the Model for insured workers or an estimate of the unemployment rate based on the average rate for the working population and on an appropriate hypothesis concerning the composition of group (i) referred to in subsection 5.2.1.

$\psi_t(h, H)$ be the reduction factors constituted by the ratio of the number of unemployed per year in receipt of benefit to the number of unemployed per year (see subsection 5.2.3 for the computation), taking into account the waiting period h and the maximum period of payment H .

The average annual number of unemployed persons in receipt of benefit will then be :

$$(5.2) \quad n_{td} = n_{ta} \cdot \lambda_t \cdot \psi_t(h, H).$$

5.2.3 Subroutine 5.1 for computation of the factors $\psi_t(h, H)$

The method of constructing the reduction factors to be used for unemployment insurance has been described in an earlier work ¹², to which

I refer the reader for an exploration of the formulae which follow, and which are based on the theory of communities subdivided into groups.

It is assumed that we know :

- (i) the unemployment rate λ_t of the insured group (see subsection 5.2.2), which is assumed to be constant in the period $t, t+1$ (which may even be shorter than one year)
- (ii) the rate ν_t at which persons enter the state of unemployment for the period $t, t+1$, i.e. the ratio of persons newly unemployed during the period to the average number of employed persons in the same period.

With regard to the latter rate it should be noted that :

- (i) The number of employed persons is supplied by the Model;
- (ii) If the number of persons newly unemployed cannot be obtained from the Model, it could be deduced from the difference between employed persons at time t and employed persons at time $t+1$ (a difference which may be plus or minus), plus the persons leaving the state of unemployment (with an appropriate estimate to allow for turnover).

It should also be noted that in practice the numbers of persons employed and of those newly unemployed needed for the computation of ν_t will not necessarily refer to the insured community, partly because the data known are often, at best, entirely general : where a rate (i.e. a ratio) is being evaluated, the approximations dictated by this difficulty are not as a rule unduly rough.

If it is now assumed that the rate of leaving the state of unemployment for whatever reason (new employment, death, emigration, etc.), i.e. the ratio of persons leaving unemployment to unemployed persons, is not a function of the duration of unemployment¹³, the coefficient $\psi_t(H)$ for a benefit with a waiting period 0 and a maximum period of payment H (primary coefficient) may be expressed by the formula

$$(5.3) \quad \psi_t(H) = 1 - e^{-\int_{t-H}^t \nu_\gamma \cdot d\gamma \cdot \lambda_\gamma}$$

The coefficient which allows for the two limits of waiting period h and maximum period of payment H is therefore provided by the difference

$$(5.4) \quad \psi_t(h,H) = \psi_t(H) - \psi_t(h)$$

5.3 Determination of the costs of unemployment benefit

The costs in question can be determined directly in a dynamic situation, as stated above.

Since the daily allowance is usually set at a fixed amount, if pd is the product of this allowance and 365, we shall have

$$(5.5) \quad \widetilde{P}_t D = \bar{n}_t d \times pd$$

5.4 Underemployment costs

These costs, which are a peculiarity of Italian legislation, depend on decisions taken from time to time by individual enterprises and vary very substantially from year to year according to the economic situation.

Consequently no codified standard procedure can be used to forecast them.

Bearing in mind that the persons involved are wage and salary earners and that it is generally the same people who are insured against unemployment, we may formulate the hypothesis that the costs are correlated with, even if somewhat in advance of the movement of the corresponding unemployment costs.

This hypothesis needs to be checked.

If the result should prove positive, the cost of this type of assistance could be determined by supposing that the number of persons in receipt of the benefit would be proportional to the unemployment costs for a succeeding period (e.g. of six months).

We observe that :

- (i) the average annual number of persons in receipt of benefit includes, as fractions of individuals, those in receipt of benefit for part of their working hours;

- (ii) the average benefit consists of the percentage of the wage prescribed by the rules, with the result that the average benefit, too, can vary over time.

In this case too the total cost is the product of the number of persons in receipt of benefit times the average benefit.

6. HEALTH CARE COSTS

6.1 General remarks

6.1.1 Forms of health care

In the developed countries, health care is now available for the entire population or most of it.

The forms of care are :

- (i) direct care, i.e. benefits in kind;
- (ii) indirect care, or reimbursement.

Health care may be organized into :

- (i) separate benefit schemes for different categories of persons receiving care;
- (ii) on single scheme, as in the case of the national health services.

6.1.2 Types of benefits

More than one type of benefit is generally provided, but they may be grouped into the following categories :

- (i) general non-hospital medical care;
- (ii) specialized non-hospital care (diagnostic and specialist services);
- (iii) pharmaceutical services;

(iv) hospital care;

(v) other benefits, generally additional (courses of thermal treatment, prostheses, etc.).

6.1.3 Computation hypothesis

The schema for computing the costs is substantially the same, whatever the form of care, the type of services and the way in which the population receiving care is subdivided into schemes.

This schema is simply based on the product : average annual number of persons receiving care x average annual cost per person receiving care.

For the sake of simplicity, however, we shall not consider the average costs per person receiving care as the product of the frequency of take-up times the average cost per service rendered.

By way of illustration, the procedure will be related to the case of a national health service, i.e. to the assumption that the whole population is insured. In dealing with the general population, we shall suppose that the numbers of persons receiving care are known (exogenously) and we shall determine the average costs in static and dynamic situation. Where there is more than one scheme, the computation must be repeated for each scheme or rather, as has been stated in relation to other benefits, for groups of schemes.

6.2 Determination of average costs

6.2.1 Static situation

We shall assume that at time 0 the following are available :

(i) the individual average annual costs

$$p_{[x]}^u, P_{[x]}^v, p_{[x]}^d, p_{[x]}^o, p_{[x]}^a$$

(ii) a distribution of persons receiving care $g[x]^a$

We would point out that, both for the costs and for the distribution, we use the symbol \bar{x} to indicate large age groups. In this type of forecasting, three groups are generally sufficient :

0-14 15-65 65 and over.

In the middle group, it may perhaps be useful to deal with each of the two sexes separately.

The total average costs are obviously obtained as simple averages. We shall therefore have :

$$(6.1) \quad \bar{p}_{ou}^* = E [p[x]^u; g[x]^a]$$

A similar method is used to determine the average values \bar{p}_{ov}^* , \bar{p}_{od}^* , \bar{p}_{oo}^* , \bar{p}_{oa}^* .

6.2.2 Dynamic situation

Given the relatively short period covered by the forecasts, we shall consider the distribution $g[x]^a$ to be constant.

Another possible reason for variations in cost lies in the increase (or in exceptional cases the decrease) in the frequency of take-up. In systems which have been in operation for some time, this factor is as a rule very insignificant, partly because some services are paid for at flat rates (capitation allowances, hourly remuneration, etc.), while others (e.g. hospital care) are often subject to budget constraints. We shall therefore confine our attention to the changes in average costs (for all ages) deriving from the change in the costs of the individual services.

In this context, there are no general criteria because increases in this sector's costs depend largely on public collective wage agreements and on the control of certain prices (e.g. of drugs); nevertheless it can be taken that :

- (i) the cost of medical care, both general and specialist, follows the trend of the remuneration of public employees;
- (ii) the cost of pharmaceutical services follows the trend of prices in the relevant productive sector;
- (iii) more than 60 per cent of the cost of hospital care depends on wages and salaries, and the remainder on the prices of food, drugs and equipment.

In the case of drugs, however it should be noted that frequency of take-up and technical innovations may be an additional dynamic factor. Let us therefore suppose that we have established, on the basis of ad hoc considerations and also by using data from the Model, the annual rates of increase in the various costs. Using ϵ to indicate the rates of change in general medical care, we shall then have, for example

$$(6.2) \quad \bar{p}_t u^* = \bar{p}_0 u^* \times (1 + \epsilon_0) \times \dots \times (1 + \epsilon_{t-1}).$$

Similar formulae hold good for the other costs.

6.3 Forecast of total costs

6.3.1 Static situation

Costs in a static situation can be determined immediately if the number n_{0a} of persons insured at time 0 is known.

Supposing that

$$(6.3) \quad \bar{p}_{0s} = \bar{p}_{0u}^* + \bar{p}_{0v}^* = \bar{p}_{0d}^* + \bar{p}_{0o}^* + \bar{p}_{0a}^*$$

we shall have

$$(6.4) \quad P_{ts} = n_{0a} \times (1 + \delta)^t \times \bar{p}_{0s}$$

where δ indicates the rate of increase in the population ¹⁴.

6.3.2 Dynamic situation

A similar formula holds good for forecasting the costs in a dynamic situation, namely :

$$(6.5) \quad \tilde{P}_{ts} = n_{0a} \times (1 + \delta)^t \times \bar{P}_{ts}$$

where for \bar{P}_{ts} a definition similar to (6.3) applies.

7. SOCIAL SECURITY CONTRIBUTIONS; OTHER REVENUE; OTHER EXPENDITURE

7.1 Contributions

The most suitable way of determining contributions would be to identify the average rate of contribution (for employers, employees and the self-employed) to be applied to wages or other income in each branch. Clearly, there will not be a single rate, but rather a set of rates, one for each category of remuneration of labour in the Model (e.g. wages; income from self-employment; or: wages of agricultural employees, wages of industrial employees, etc., income from self-employment in agriculture etc.).

However, legal provisions vary widely over time and over space, which makes it difficult to give a sufficiently general presentation of these mechanisms. Moreover, the representation of contributions in most of the econometric models observed is substantially similar to that described above. Consequently, for the sake of simplicity, we shall not deal with the matter here; the reader is referred to the relevant sources, and reminded that the subject is important even if it is not studied in detail here.

7.2 Other revenue; other expenditure

For the sake of simplicity again, we do not deal here with contributions and costs for occupational accidents and diseases, with revenue and expenditure on temporary invalidity allowances, or with other revenue and expenditure that are required for a complete picture of social security activity, and may therefore be essential to some types of analysis.

8. REDISTRIBUTION EFFECT

8.1 General remarks

8.1.1 Contents of this chapter

As we have already pointed out (see subsection 1.2.1), one of the specific reasons for constructing the MASS was to perform more detailed analyses, first of all the redistribution effect.

This effect can be determined only when changes occurring (or assumed) in the social security system are substantial enough to generate appreciable variations in the structure of incomes and consequently in households' propensity to save.

The general method for incorporating the redistribution effect may be summarized as follows: on the basis of a distribution of households by income bracket (using disposable income, i.e. net of taxes and social security contributions payable before the changes were made), we calculate a new distribution that would presumably result from the assumed changes in contributions, taxes or benefits. We then apply to each of the two distributions the coefficient of propensity to save for each income bracket, obtaining two average coefficients; the ratio between them is the change in the relevant average coefficient to be applied to the Model.

It is important to note that this procedure based on income brackets takes account, not of individual households, but of groups of households; consequently the redistribution effect is not fully apparent, for part of it is ignored.

8.1.2. Introductory remarks

In the pages which follow, references to social security contributions should be understood to mean only those of employees and of the self-employed.

Tax changes introduced with changes in social security rules are taken into account in relation to each income bracket when the changes affect direct taxes, and strictly proportionally when they affect indirect taxes.

Increases in health care benefits are not taken into consideration because in the industrialized countries they nearly always correspond to an improved service and do not appreciably change disposable income. Moreover if they are granted in kind, as is often the case, they come under the heading of public consumption. Where the

changes involve extending protection to households not previously covered, however, the propensity to save is necessarily affected because the change eliminates the need for expenditure that the households concerned would otherwise have incurred.

However, it was not thought necessary to complicate the issue by taking account of this case, which is of limited application under present arrangements.

We do, on the other hand, consider changes in contributions and taxes associated with changes in health care benefits.

Since changes in the rules (whether actual or assumed) are non-recurring events, there is no need to consider them from a dynamic point of view; our interest here is in estimating how the propensity to save is modified as a result of such changes. The variation in the propensity to save can be utilized for the year when the new rules are introduced and for the following year, and we can apply it to the coefficient of propensity to save deriving from the forecasts provided by the Model for changes in income in general.

8.2 Basic data and assumptions

8.2.1 Basic data

Although the statistics available in income in Italy are poor by comparison with those available for other countries, it is possible, using a set of data¹⁵, very small samples and some estimates and interpolations, to obtain :

- (i) a two-tier distribution of households by aggregate income and by transfer income (which is almost entirely composed of social insurance benefits);
- (ii) for each class of aggregate income, the average percentage accounted for by income from capital, income from employment other than self-employment and, obviously, transfer income, the remainder being termed "other income" (mainly income from self-employment).

In deriving this two-tier distribution from the available statistics, some adjustments are required.

First, any transfer that do not consist of social insurance benefits should be eliminated 16.

We referred above to a loss of part of the redistribution effect; to attenuate this loss, the two-tier distribution should cover at least 15 or 20 aggregate income brackets, obtained on the basis of appropriate interpolations. Let m be the number of income brackets. Further, let n be the number of transfer-income subclasses ($5 \leq n \leq 10$). The lowest transfer-income subclass comprises households with no transfer income.

The distribution should be constructed in such a way as to avoid obvious incongruities. For example, when an aggregate income bracket is split in two and the households are assigned to subgroups as a function of the distribution of social insurance benefits, care should be taken to ensure that the transfer income of the smaller class does not exceed its aggregate income.

It may also be necessary in the lower brackets to have fewer than n classes of social insurance income.

8.2.2 Main assumptions

In the absence of precise information to the contrary, the following assumptions are made :

- (i) In each of the m income brackets, the residual income left after deduction of the n classes of social insurance income from the aggregate income of the households in the bracket is assumed to be made up of the same percentages of income from capital, income from employment other than self-employment and other income as those observed in the aggregate income bracket considered.
- (ii) In each of the n classes of social insurance income, the relevant income is further subdivided into pensions, on the one hand, and other benefits (sickness benefits, unemployment benefits, family

allowances, etc.) on the other; unless a more suitable method or precise information on this breakdown is available, it is assumed that it corresponds to the breakdown, at the date of the calculation, of total expenditure on social security benefits into pensions (as a percentage of the total) and other benefits ¹⁷.

Both these assumptions are largely arbitrary, but more suitable ones are difficult to formulate without appropriate and detailed statistics.

8.3 Determining the redistribution effect

8.3.1 Increase (or reduction) in benefits with no change in entitlement

On the basis of the data and assumptions described in section 8.2, all the data available at the time of calculation can be brought together:

- (i) in a row vector with (m x n) elements

$$\hat{f} = [f(h,k)]$$

where $f(h,k)$ is the number of households with aggregate income in class h and social insurance income in class k;

- (ii) in a matrix of (m x n) rows and 5 columns

$${}^{\circ}R_f = [r(h,k)j]$$

where $r(h,k)j$ is the income from source j of each household in group $f(h,k)$, and where j is one of the five sources of income considered (income from capital, income from employment other than self-employment, other income from employment, social insurance pensions, other social insurance benefits);

- (iii) in a row vector with (m x n) elements

$$o_u = [u(h,k)]$$

where the elements are coefficients of propensity to save as a function of aggregate income (there will be only m different values).

Whenever it is though necessary to determine how a change in the rules - financing arrangements, changes in benefits, or both - will affect the propensity to save, a matrix with (m x n) rows and 5 columns can be constructed:

$$\hat{\phi} = [\phi(h,k)j]$$

Where the elements are the coefficients to be applied to the values in the (m x n) rows of matrix 0R_f for the 5 categories of income; these coefficients may determine an increase (coefficient > 1), a decrease (coefficient < 1) or no change (coefficient = 1).

For example, taking a group of households where the average annual income is:

income from capital	LIT 1.000.000
income from employment other than self-employment	LIT 11.000.000
income from self-employment	LIT 4.000.000
income from pensions	LIT 3.200.000
income from other benefits	LIT 800.000

an increase of 10 per cent in pensions financed by an increase in contributions of 1 per cent of wages and salaries and 0.50 per cent of the income of the self-employed will give the following row corresponding to an aggregate income of LIT 20 million and a social insurance income of LIT 4 million in matrix

$$\left| \begin{array}{c} 1; 0.99; 0.995; 1.10; 1 \end{array} \right|$$

The simple product matrix

$$(8.1) \quad {}^1\hat{R}_f = {}^0\hat{R}_f \cdot \hat{x} \cdot \hat{\phi}$$

describes the situation of these households' incomes after the change.

To show how the propensity to save has changed, all that now needs to be done is to introduce a new row vector

$${}^1\hat{u} = \left[u(n, k) \right]$$

corresponding to the new incomes resulting from the sum of the rows in matrix ${}^1\hat{R}_f$; and to carry out the following operation:

$$(8.2) \quad \bar{u} = E \left[\hat{u}; \hat{f}' \cdot \hat{x} \cdot ({}^0\hat{R}_f \hat{x} \hat{e}') \right] = \frac{\bar{u}_x / \hat{f}' \cdot \hat{x} \cdot ({}^0\hat{R}_f \hat{x} \hat{e}')}{\bar{e}_x / \hat{f}' \cdot \hat{x} \cdot ({}^0\hat{R}_f \hat{x} \hat{e}')}$$

$$(8.3) \quad {}^1\bar{u} = E \left[{}^1\hat{u}; \hat{f}' \cdot \hat{x} \cdot ({}^1\hat{R}_f \hat{x} \hat{e}') \right] = \frac{{}^1\bar{u}_x / \hat{f}' \cdot \hat{x} \cdot ({}^1\hat{R}_f \hat{x} \hat{e}')}{\bar{e}_x / \hat{f}' \cdot \hat{x} \cdot ({}^1\hat{R}_f \hat{x} \hat{e}')}$$

$$(8.4) \quad \gamma = \frac{{}^1\bar{u}}{\bar{u}}$$

In the notation adopted, \hat{e}' is the unit column vector with 5 rows, \hat{e} the row vector with $(m \times n)$ columns, $0\bar{u}$ and $1\bar{u}$ the average coefficients of propensity to save before and after application of the new rules, and Δ the change in the average coefficient to be applied to the Model.

8.3.2 Increase in benefits corresponding to an extension of entitlement

When the change in the rules entails including a new segment of the population in the group of entitled persons, with a consequent increase in expenditure and possibly in contributions also, the procedure described in subsection 7.3.1 can be applied. However, this approach ignores the probability that the number of households with no social insurance income will be reduced, i.e. it does not take account of the change in the distribution recorded in vector \hat{f} .

For there is a high probability that, while some of the newly entitled persons belong to households already in receipt of income from social insurance benefits, other will belong to households previously included in groups $f(h, 0)$. It is not easy to estimate the reduction in the number of households receiving no transfer income (which corresponds of course to an increase in the population of other classes, assumed to be distributed proportionally between them).

Consequently, a case-by-case examination is required on the basis of the arrangements in force in the country considered and the features of the newly-entitled population¹⁸

This leads to the construction of a new vector f . This new vector is introduced into relationship (8.3), while relationship (8.2) remains unchanged.

For the coefficient of increase in matrix $\hat{\Phi}$ for the social insurance income classes considered, care should be taken to see that the overall social insurance income is equal to the original figure plus the increase due to the extension of entitlement. This should be fairly easy to do by trying out one or two possibilities, without any need for complicated formulae¹⁹.

8.3.3 Other cases

Although there are many ways in which social security rules can be changed, they all in fact correspond in the final analysis to the changes dealt with in subsection 8.3.1 and 8.3.2. For example, a change in the system of financing with no change in benefits can be accounted for using the method described in subsection 8.3.1: appropriate coefficients are included in matrix for the first three types of income, and coefficients of 1 for the other two.

An increase in expenditure on benefits due neither to an increase in individual benefits nor to an extension of entitlement, but to more frequent occurrence of the event giving rise to receipt of the benefit (notably unemployment) is not considered here; as we said at the outset, the redistribution effect is mainly seen when the rules are substantially changed. This may mean that new laws are under consideration, or simply that we want to test the probable effects of reforms that could be introduced.

8.4 Additional comments

In subsection 8.2.2 we assumed that social insurance income was made up of two parts: pensions and other benefits. This subdivision is justified by the basic difference between pensions, usually paid to insured persons who have withdrawn from the labour market, and other benefits, usually received by members of the labour force.

However, it is also possible to subdivide social insurance income into three parts: pensions, family allowances and other benefits. This would be justified if information were available to determine percentages for each of three categories and for each class of aggregate income. There is no problem about adjusting the procedure described above to take account of the finer breakdown.

Lastly, it is worth noting that the calculations we have described can be applied to specific types of households (e.g. households whose head is a wage or salary earner), either when the Model is used to consider such types of households separately, or simply for information.

9. APPLICATION OF THE SUGGESTED PROCEDURE : AN EXAMPLE

9.1 Conditions of the experiment

The description of this system could not be concluded without at least partial empirical testing of the proposed methods, to check on their feasibility. Provisional estimates have therefore been drawn up for all the parameters needed to apply the algorithm concerning the number of pensioners, presented in Chapter 2 above. The procedure was then linked to the model for Italy drawn up by the Associazione Prometeia. To facilitate the link-up, the matrices \hat{Q}_p of elimination/remaining have been broken down into linear equations²⁰, so as to provide the results of matrix multiplications column by column (except for the first column). This step will be required whenever the data-processing method used is unsuitable for resolving dynamic linear systems that also involve matrix algebra, such as the Troll method.

Thus, the Italy model gives an initial projection to 1983-88 without the proposed algorithm; the variable "total number of pensions" thus remains exogenous, i.e. estimated outside the model.

The equations for the suggested method were then added at the end of the model, and the variable "pensions" became endogenous. The results obtained in the two ways were then compared for the most representative magnitudes.

9.2 Findings

The comparisons are summarized in Tables A, B, C and D above, for each period of simulation. In the first column appear the results of using the model without the method suggested here; in the second column appear the results with the method. The third and fourth columns show, respectively, the absolute and percentage differences between results without and with the method.

- Effect of insertion of the pensions algorithm.

A. NUMBER OF PENSIONS

		Exogenous Pensions	ENDOGENOUS PENSIONS		
		(1)	(2)	(3)=(2)-(1)	(4)=100.(3)/(1)
1983	1	16779.50	16689.60	-89.89	-0.54
1983	2	16779.50	16797.90	18.36	0.11
1983	3	16779.50	16905.50	125.99	0.75
1983	4	16779.50	17012.60	233.14	1.39
1984	1	17248.60	17119.40	-129.24	-0.75
1984	2	17248.60	17226.30	-22.31	-0.13
1984	3	17248.60	17332.90	84.33	0.49
1984	4	17248.60	17439.30	190.73	1.11
1985	1	17679.40	17545.40	-133.99	-0.76
1985	2	17679.40	17651.60	-27.83	-0.16
1985	3	17679.40	17757.40	77.97	0.44
1985	4	17679.40	17862.80	183.40	1.04
1986	1	18174.50	17967.90	-206.61	-1.14
1986	2	18174.50	18073.00	-101.55	-0.56
1986	3	18174.50	18177.70	3.20	1.75813E-02
1986	4	18174.50	18282.20	107.68	0.59
1987	1	18627.90	18386.40	-241.52	-1.30
1987	2	18627.90	18490.40	-137.50	-0.74
1987	3	18627.90	18594.20	-33.74	-0.18
1987	4	18627.90	18697.70	69.79	0.37
1988	1	19075.00	18801.00	-274.00	-1.44
1988	2	19075.00	18904.20	-170.79	-0.90
1988	3	19075.00	19007.10	-67.86	-0.36
1988	4	19075.00	19109.90	34.90	0.18

B. SOCIAL TRANSFERS (billion lire)

		Exogenous Pensions	ENDOGENOUS PENSIONS		
		(1)	(2)	(3)=(2)-(1)	(4)=100.(3)/(1)
1983	1	23925.80	23832.00	-93.84	-0.39
1983	2	24750.00	24770.10	20.15	8.14080E-02
1983	3	26097.10	26243.50	146.36	0.56
1983	4	27260.90	27542.80	281.95	1.03
1984	1	28809.30	28643.10	-166.23	-0.58
1984	2	29421.70	29392.50	-29.17	-0.10
1984	3	29461.00	29575.40	114.36	0.39
1984	4	30999.70	31271.20	271.48	0.88
1985	1	33041.50	32840.90	-200.64	-0.61
1985	2	33535.50	33494.80	-40.69	-0.12
1985	3	34671.30	34799.00	127.65	0.37
1985	4	35953.20	36259.90	306.70	0.85
1986	1	37684.80	37324.30	-360.52	-0.96
1986	2	38448.30	38270.50	-177.84	-0.46
1986	3	39867.90	39879.00	11.12	2.78851E-02
1986	4	40852.70	41062.00	209.32	0.51
1987	1	42717.30	42232.20	-485.07	-1.14
1987	2	43679.90	43400.10	-279.71	-0.64
1987	3	44933.60	44863.50	-70.10	-0.16
1987	4	46250.60	46401.20	150.51	0.33
1988	1	48671.70	48034.30	-637.41	-1.31
1988	2	49711.00	49307.50	-403.50	-0.81
1988	3	51085.60	50917.00	-168.63	-0.33
1988	4	52682.80	52760.70	77.91	0.15

C. DEFICIT OF THE PUBLIC AUTHORITIES (billion lire)

		Exogenous Pensions	ENDOGENOUS PENSIONS		
		(1)	(2)	(3)=(2)-(1)	(4)=100(3)/(1)
1983	1	-13112.70	-13016.20	96.55	-0.74
1983	2	-17442.20	-17464.90	-22.70	0.13
1983	3	-14665.00	-14813.40	-148.42	1.01
1983	4	-12989.00	-13277.40	-288.41	2.22
1984	1	-17426.60	-17254.00	172.62	-0.99
1984	2	-22286.90	-22267.50	19.41	-8.70745E-02
1984	3	-17963.00	-18083.40	-120.39	0.67
1984	4	-15357.80	-15635.60	-277.77	1.81
1985	1	-18548.50	-18336.90	211.64	-1.14
1985	2	-24679.70	-24653.30	26.39	-0.11
1985	3	-19980.60	-20113.50	-132.96	0.67
1985	4	-15380.50	-15693.70	-313.23	2.04
1986	1	-19958.10	-19587.00	371.11	-1.86
1986	2	-28303.00	-28150.10	152.82	-0.54
1986	3	-23534.80	-23556.50	-21.70	9.22171E-02
1986	4	-17881.20	-18098.40	-217.20	1.21
1987	1	-22737.50	-22250.50	487.08	-2.14
1987	2	-33149.70	-32905.30	244.43	-0.74
1987	3	-26862.30	-26804.90	57.34	-0.21
1987	4	-19803.10	-19962.60	-159.53	0.81
1988	1	-26227.50	-25593.40	634.13	-2.42
1988	2	-37571.80	-37207.90	363.93	-0.97
1988	3	-31380.90	-31223.00	157.88	-0.50
1988	4	-22618.80	-22700.50	-81.75	0.36

D. BALANCE OF CURRENT PAYMENTS (billion lire)

		Exogenous Pensions	ENDOGENOUS PENSIONS		
		(1)	(2)	(3)=(2)-(1)	(4)=100(3)/(1)
1983	1	-5571.73	-5562.75	8.98	-0.16
1983	2	-1708.93	-1703.91	5.02	-0.29
1983	3	1795.64	1790.01	-5.63	-0.31
1983	4	2476.50	2450.01	-26.48	-1.07
1984	1	-5943.20	-5950.64	-7.45	0.13
1984	2	-2770.17	-2781.12	-10.95	0.40
1984	3	-203.96	-221.20	-17.24	8.45
1984	4	1983.03	1939.52	-43.51	-2.19
1985	1	-4462.96	-4476.07	-13.11	0.29
1985	2	-998.58	-1013.99	-15.41	1.54
1985	3	-344.69	-364.38	-19.69	5.71
1985	4	243.22	191.48	-51.74	-21.27
1986	1	-2664.53	-2662.83	1.70	-6.37718E-02
1986	2	-864.49	-852.71	11.78	-1.36
1986	3	-855.67	-836.24	19.43	-2.27
1986	4	-526.02	-534.80	-8.77	1.67
1987	1	-3246.00	-3196.51	49.48	-1.52
1987	2	-1867.10	-1805.53	61.57	-3.30
1987	3	-1502.82	-1430.24	72.58	-4.83
1987	4	-914.50	-872.73	41.77	-4.57
1988	1	-4140.19	-4029.71	110.48	-2.67
1988	2	-3117.17	-2989.82	127.36	-4.09
1988	3	-3005.00	-2862.94	142.06	-4.73
1988	4	-2519.40	-2410.95	108.45	-4.30

This shows that the movement in the number of pensioners and thus in the levels tends with the new specification to accelerate in the years 1983, 1984 and 1985 and then to decelerate slightly compared with the reference simulation. Taking the average of the respective annual data we obtain the following results in thousands of persons.

	1983	1984	1985	1986	1987	1988
1) No. of exogenous pensions	16,780	17,250	17,680	18,175	18,624	19,075
2) No. of endogenous pensions	16,851	17,280	17,705	18,125	18,542	18,955
3) Difference = 2 - 1	+71	+30	+25	-50	-80	-120

These data make it clear that if the movement in the number of pensioners is evaluated on the basis of insured persons and not on the basis of persons of pensionable age, the qualitative estimates which are at present being made slightly underestimate the number of pensioners until the year 1985 and over-estimate it subsequently. It is of course true that this prospect would change if the economic development towards the 1988 horizon, and in particular the movement in employment, were to change substantially.

Furthermore, given the very much reduced level of the differences between the two series in this exercise, which has really been done by way of an example, the effects of the other economic quantities remain marginal. Finally, it is worth noting that while the experiment was being carried out one noted a great stability of the results, which do not lead to any difficulty at all in respect of convergence of the model.

10. SOME CONCLUSIONS OF THE PROBLEM OF SPECIFICATION

The examination that we have made in the above paragraphs and above all the examples put forward justify some conclusions on the problem of specification of social security.

First: One must recognise that social security is a mechanism which is very complex and has such special characteristics that it cannot be represented by a few equations included in a general model.

Second: The construction of auxiliary models enables one to describe this mechanism adequately and in all the detail that is considered useful for:

- transmitting to the general model trustworthy projections of entries and departures;
- acquiring additional elements of information, which can also be used for subsequent support to the general model.

Third: The auxiliary model permits the appropriate use of algorithms which are more complex than those that can be expressed by linear relations based on regression. In particular it makes possible to use "distributions" instead of "average values". This occurs without any particularly significant methodological difficulties.

Fourth: From the exchange of information between the general and the auxiliary models one can also improve the forecasts of the future development of social security, considered independently.

In this light it seems that this is the appropriate point at which to invite the public institutions to intensify efforts directed to the planning and production of statistics which will be suitable to meet the new requirements.

*

* *

Appendix: NOTATION

1. Preliminary remarks

The notation described here has been devised not only for the purposes of this paper but also for the more general purpose of expressing the formulae in the MASS in order to apply them to a specific country.

Consequently, in the context of this paper, some of the conventions and symbols will seem too detailed or redundant.

2. Conventions

I. The symbols referring to the different magnitudes consist as a rule of one or two letters and an Arabic numeral with no intervening space or sign.

The first letter indicates the nature of the magnitude; the second indicates the activity or type of pension concerned; the number indicates the scheme.

II. As a rule each magnitude is identified by a combination of two letters and a number. When the subject is being dealt with in general terms, the number and, where appropriate, the second letter may be omitted. Consequently, when a magnitude is referred to by a single letter; this should be understood to be the first letter of the symbol. If the number is omitted, the magnitude in question covers all schemes unless it has been specifically stated that only one is concerned.

III. Indices may be associated with individual letters to specify further the type of magnitude, the date, age, etc.

IV. Multiplication signs are always explicit

V. Upper case letters are used for the output of the MASS. When magnitudes are considered from the dynamic point of view, a swung dash is placed above the corresponding letter to distinguish it from the magnitude considered from the static point of view.

(Appendix)

VI. As a rule, Greek letters are used for parameters, coefficients and rates.

VII. Averages are referred to by using the symbol \bar{E} followed by an indication of quantities and weights in brackets, or by placing a dash - above the symbol.

VIII. Matrices and vectors are identified by upper case and lower case letters respectively, in heavy type.

IX. Matrices, vectors, parameters, coefficients, rates and a few symbols of limited application are defined in the text as required. The significance of the symbols may change from one chapter to another.

(Appendix)

3. Main symbols

The output of the MASS

1st symbol: type of magnitude

W: wages

C: employers' contributions

D: employees' contributions

B: self-employed workers' contributions

P: benefits

U: other revenue

in particular:

G: government grant

I(+): interest received

T(+): transfers from other accounts

V: other expenditure

in particular:

I(-): interest paid

T(-): transfers to other accounts

A: administrative expenditure

S: expenditure on asset management

R: reserves or balance on assets

2nd symbol: branch

P: pensions

B: family allowances

M: temporary incapacity

D: unemployment

S: health care

L: relevant events

(Appendix)

If a magnitude relates to more than one branch, the second symbol consists of two or more letters underlined thus: MS.

If a magnitude relates to all branches, the second symbol is omitted.

3rd symbol: scheme

Arabic numerals 1, 2, 3, etc....

The numeral 1 is reserved for the general scheme. If the magnitude relates to more than one scheme, the numbers are underlined thus: 1.3.

If there is only one scheme, or if the reference is to all schemes, the third symbol is omitted.

Other magnitudes (MASS inputs, inputs and outputs of subroutines)

1st symbol: type of magnitude

n: annual average (absolute value)

w: annual per capita wage (absolute value)

c: annual per capita contribution (absolute value)

p: annual per capita benefit (absolute value)

q: probability of change from one status to another, probability of elimination, frequency of a specific event

g: distribution of a group of individuals according to a specific characteristic (age, family situation, income bracket, etc....)

To represent the probability of change from one status to another, the symbols for the two statuses are given, separated by a stroke and preceded by the symbol q thus: q a/i (= the probability of invalidity for a member of the labour force). The letters separated by a stroke are to be taken together and count as the second symbol.

2nd symbol: status or branch

a: the labour force or insured persons

p: pensions

in particular:

i: invalidity pension

v: old-age pension

f: survivors pension

b: family allowances

m: temporary incapacity

s: health care

in particular:

u*: general medical aid

v*: specialist medical aid

d*: pharmaceutical aid

o*: hospital aid

a*: other aid

d: unemployment

l: relevant events

in particular:

i*: permanent invalidity

f*: survivors

m*: temporary incapacity

3rd symbol: scheme

Arabic numerals 1, 2, 3.

Where more than one branch is involved, or where the general scheme, several schemes or all schemes are referred to, the same conventions apply as to the second symbol for the output magnitudes of the MASS.

(Appendix)

Other symbols in common use

j, h and k are usually used to indicate economic sectors.

x and t are used to indicate age and time respectively (e.g. $g_{x,t}$ is the age distribution at time t).

f is used to indicate the situation of households (e.g. g_f is the distribution by number of dependants).

σ is the annual rate of inflation

ω is the annual increase in real incomes or wages

$$\rho = \sigma + \omega$$

δ is the annual increase or decrease in the population or the labour force.

4. Simple product of matrices and vectors

For the purposes of this work, in addition to the usual definition of the product of two vectors or matrices, we have introduced the notion of a simple product symbolized thus : \hat{X} .

Given two row (or column) vectors of m elements

$$\hat{a}_m = [\hat{a}_j] = \left\| \begin{array}{cccc} a_1 & a_2 & \dots & a_m \end{array} \right\|$$

$$\hat{b}_m = [\hat{b}_j] = \left\| \begin{array}{cccc} b_1 & b_2 & \dots & b_m \end{array} \right\|$$

we call "simple product" the row (or column) vector of m elements

$$\hat{C}_m = [\hat{a}_j \times \hat{b}_j] = \left\| \begin{array}{cccc} a_1 \times b_1 & a_2 \times b_2 & \dots & a_m \times b_m \end{array} \right\|$$

and we write

$$(A.1) \quad \hat{C}_m = \hat{a}_m \hat{x} \hat{b}_m$$

(Appendix)

Similarly, given two matrices of m rows and n columns :

$$\hat{A}_{m,n} = [a_{j,h}]$$

$$\hat{B}_{m,n} = [b_{j,h}]$$

their simple product is matrix

$$\hat{C}_{m,n} = [a_{j,h} \times b_{j,h}]$$

also containing m rows and n columns; we note

$$(A.2) \quad \hat{C} = \hat{A} \times \hat{B}$$

5. Matricial averages

Given two $m \times n$ matrices $\hat{A}_{m,n}$ and $\hat{B}_{m,n}$, we may calculate the weighted average of the elements of $\hat{A}_{m,n}$ using the elements of $\hat{B}_{m,n}$ as weights, thus:

$$(A.3) \quad E(\hat{A}_{m,n}; \hat{B}_{m,n}) = \frac{\hat{e}_m \times [\hat{A}_{m,n} \times \hat{B}_{m,n}] \times \hat{e}'_n}{\hat{e}_m \times \hat{B}_{m,n} \times \hat{e}'_n}$$

where \hat{e}_m and \hat{e}_n are unit line vectors of m and n elements respectively.

In particular, given two row vectors \hat{a}_m and \hat{b}_m (each containing m elements), the average of the values of the elements of \hat{a}_m weighted for the elements of \hat{b}_m is calculated using the following formula:

$$(A.4) \quad E(\hat{a}_m, \hat{b}_m) = \frac{[\hat{a}_m \times \hat{b}_m] \hat{e}'_m}{\hat{b}_m \times \hat{e}'_m} = \frac{\hat{a}_m \times \hat{b}_m}{\hat{b}_m \times \hat{e}'_m}$$

If two column vectors are used, the calculations are analogous.

In the text, the subscripts for the number of rows and/or columns in the vector \hat{e} and the other vectors and matrices used are omitted for the sake of simplicity.

(Appendix)

6. Abbreviations and references

MASS: Auxiliary model for the social security system

Model: Main model for the economy of a given country

Sbr: Subroutine

References are made as follows:

- (i) in references to the text, the number of the relevant chapter is followed by one or two other numbers indicating the section or the subsection;
- (ii) in references to formulae, the number of the relevant chapter is followed by the number identifying the formula, the whole expression being in brackets;
- (iii) in references to subroutines, the number of the chapter is followed by a number indicating the position of the relevant subroutine in the order given in the chapters (i.e. 1 for the first subroutine given, 2 for the second, etc. ...);
- (iv) in references to footnotes, the number of the chapter is followed by the footnote number, the whole expression being in brackets.

Footnotes:

- 1 See M.A. Coppini, "Perspectives d'utilisation des Modèles économétriques pour l'étude des problèmes du financement de la sécurité sociale". The prospects for using econometric models to study the problems of social security financing, Geneva 1981 (in course of publication). The Italian text has been published in issue n° 126 of Istituto di Scienze Attuariali of the University of Rome (1982). This report deals with some applications made with the models PROMETEIA (Italy) and METRIC (France). The French text will be published together with a study by G. Laina, "The economic effects of changing the structure of financing social security systems", Brussels, 1982. In this study, analogous applications are extended to four countries (Italy, France, the Federal Republic of Germany and the United Kingdom); the EUROLINK procedure is used to link the economies of these countries appropriately, and both the direct effects on the economy of the country adopting specific measures and the indirect effects on the other countries are computed. See the documents of the Geneva meeting.
- 2 See M.J. Festjens and P. Hüge, "SOZEC - Un module prévisionnel des comptes par branche de la sécurité sociale (A forecasting model of the accounts by branch of the social security system). In Revue belge de la sécurité sociale, 1980.
- 3 The age will be that of the oldest survivor or of the insured person or pensioner on whose account the pension is paid.
- 4 The probabilities in question are usually selected. In order to avoid complex computations, certain assumption will be made as to the "complementary duration" of each of the groups making up the distribution. See L. Crescentini for possible examples of the use of selected tables.
- 5 See R. Spallucci, "Sui metodi di calcolo dell'annualità di famiglia (Methods of computing annual family income), Istituto di Scienze Attuariali of the University of Rome, issue n° 27, 1972. The comments made in footnotes ¹ and ² also hold good for these probabilities.
- 6 The employment of the matrix calculation for demographic projections in pensions matters is well known. Cf. M.A. Coppini, Lezioni di tecnica delle assicurazioni sociali (Lectures on the technique of social insurance). Published Veschi, Rome, 5th edition, 1979. For determination of the average rates contained in the matrices shown under (5), Cf. L. Crescentini "Una semplificazione per il calcolo di valori attuali medi con le tavole selezionate (A simplification for the calculation of current average values using selective tables). To be published shortly.
- 7 The rules in Italy preclude this case.
- 8 If the legislation remains unchanged, the average pensions paid in certain years preceding the computation may, after the dynamic factors have been eliminated, provide useful information.
- 9 Including heads of households who have no dependent family.

- 10 Including zero benefit for heads of households who have no dependants
- 11 See M.A. Coppini, "Lezioni di tecnica delle assicurazioni sociali" (Lessons in social insurance technique), 7th edition, Eredi Veschi, Chapter XIII, page 218.
- 12 See M.A. Coppini, "A propos des méthodes d'évaluation des charges de l'assurance sociale contre le chômage" (Methods of evaluating unemployment insurance costs) in Atti della II Conferenza degli Attuari e degli Statistici della Sicurezza Sociale, (Proceedings of the second conference of social security actuaries and statisticians, Rome, 1959.
- 13 It should be noted that it is not necessary to know this rate, but only to be able to rely on the hypothesis.
- 14 The reader is reminded that we have assumed the existence of a national health service.
- 15 See "Reddito, risparmio e patrimonio immobiliare delle famiglie italiane nell'anno 1979" (Income, savings and immovable property of Italian families in 1979), Bollettino della Banca d'Italia, 35th year, n°s 3 and 4.
- 16 A two-tier distribution of this type has been constructed by F. Spandonaro on the basis of the data referred to in note to this chapter. Cfr. F. Spandonaro : "La distribuzione delle famiglie italiane rispetto al reddito totale e previdenziale : una proposta di calcolo". In corso di stampa.
- 17 Health service expenditure should be deducted from the total if it is not expenditure on transfer payments.
- 18 E.g. if entitlement to unemployment benefits is extended to a segment of the population that was already entitled to sickness benefits, there will probably be no reduction in the number of households in groups f_{h0} .
- 19 If expenditure is increased by e.g. 20 per cent as a result of the extension of entitlement to unemployment insurance, the coefficient of increase will be smaller than 1.2 because there has also been an increase in the number of families with social insurance income.
- 20 Nine equations in all (three for each matrix), plus a tenth equation summing the findings, to be used in the main model in an iterative process.

Economic Papers

The following papers have been issued. Copies may be obtained by applying to the address mentioned on the inside front cover.

- No. 1 EEC-DG II inflationary expectations. Survey based inflationary expectations for the EEC countries, by F. Papadia and V. Basano (May 1981).
- No. 3 A review of the informal economy in the European Community, by Adrian Smith (July 1981).
- No. 4 Problems of interdependence in a multipolar world, by Tommaso Padoa-Schioppa (August 1983).
- No. 5 European Dimensions in the Adjustment Problems, by Michael Emerson (August 1981).
- No. 6 The bilateral trade linkages of the Eurolink Model : An analysis of foreign trade and competitiveness, by P. Ranuzzi (January 1982).
- No. 7 United Kingdom, Medium term economic trends and problems, by D. Adams, S. Gillespie, M. Green and H. Wortmann (February 1982).
- No. 8 Où en est la théorie macroéconomique, par E. Malinvaud (juin 1982).
- No. 9 Marginal Employment Subsidies : An Effective Policy to Generate Employment, by Carl Chiarella and Alfred Steinherr (November 1982).
- No. 10 The Great Depression : A Repeat in the 1980s ?, by Alfred Steinherr (November 1982).
- No. 11 Evolution et problèmes structurels de l'économie néerlandaise, par D.C. Breedveld, C. Depoortere, A. Finetti, Dr. J.M.G. Pieters et C. Vanbelle (mars 1983).
- No. 12 Macroeconomic prospects and policies for the European Community, by Giorgio Basevi, Olivier Blanchard, Willem Buiter, Rudiger Dornbusch and Richard Layard (April 1983).
- No. 13 The supply of output equations in the EC-countries and the use of the survey-based inflationary expectations, by Paul De Grauwe and Mustapha Nabli (May 1983).

- No. 14 Structural trends of financial systems and capital accumulation : France, Germany, Italy, by G. Nardozi (May 1983).
- No. 15 Monetary assets and inflation induced distortions of the national accounts - conceptual issues and correction of sectoral income flows in 5 EEC countries, by Alex Cukierman and Jorgen Mortensen (May 1983).
- No. 16 Federal Republic of Germany. Medium-term economic trends and problems, by F. Allgayer, S. Gillespie, M. Green and H. Wortmann (June 1983).
- No. 17 The employment miracle in the US and stagnation employment in the EC, by M. Wegner (July 1983).
- No. 18 Productive Performance in West German Manufacturing Industry 1970-1980; A Farrell Frontier Characterisation, by D. Todd (August 1983).
- No. 19 Central-Bank Policy and the Financing of Government Budget Deficits : A Cross-Country Comparison, by G. Demopoulos, G. Katsimbris and S. Miller (September 1983).
- No. 20 Monetary assets and inflation induced distortions of the national accounts. The case of Belgium, by Ken Lennan (October 1983).
- No. 21 Actifs financiers et distorsions des flux sectoriels dues à l'inflation : le cas de la France, par J.-P. Baché (octobre 1983).
- No. 22 Approche pragmatique pour une politique de plein emploi : les subventions à la création d'emplois, par A. Steinherr et B. Van Haeperen (octobre 1983).
- No. 23 Income Distribution and Employment in the European Communities 1960 - 1982, by A. Steinherr (December 1983).
- No. 24 U.S. Deficits, the dollar and Europe, by O. Blanchard and R. Dornbusch (December 1983).
- No. 25 Monetary assets and inflation induced distortions of the national accounts. The case of the Federal Republic of Germany, by H. Wittelsberger (January 1984).
- No. 26 Actifs financiers et distorsions des flux sectoriels dues à l'inflation : le cas de l'Italie, par A. Reati (janvier 1984).
- No. 27 Évolution et problèmes structurels de l'économie italienne, par Q. Ciardelli, F. Colasanti et X. Lannes (janvier 1984).
- No. 28 International Co-operation in Macro-economic Policies, by J.E. Meade (February 1984).

- No. 29 The Growth of Public Expenditure in the EEC Countries 1960-1981 : Some Reflections, by Douglas Todd (December 1983).
- No. 30 The integration of EEC qualitative consumer survey results in econometric modelling : an application to the consumption function, by Peter Praet (February 1984).
- No. 31 Report of the CEPS Macroeconomic Policy Group. EUROPE : The case for unsustainable growth, by R. Layard, G. Basevi, O. Blanchard, W. Buiter and R. Dornbusch (April 1984).
- No. 32 Total Factor Productivity Growth and the Productivity Slowdown in the West German Industrial Sector, 1970-1981, by Douglas Todd (April 1984).
- No. 33 An Analytical Formulation and Evaluation of the Existing Structure of Legal Reserve Requirements of the Greek Economy : An Uncommon Case, by G. Demopoulos (June 1984).
- No. 34 Factor Productivity Growth in Four EEC Countries, 1960-1981 by Douglas Todd (October 1984).
- No. 35 Rate of profit, business cycles and capital accumulation in U.K. industry, 1959-1981, by Angelo Reati (November 1984).
- No. 36 Report of the CEPS Macroeconomic Policy Group. Employment and Growth in Europe : A Two-Handed Approach by P. Blanchard, R. Dornbusch, J. Drèze, H. Giersch, R. Layard and M. Monti (June 1985).
- No. 37 Schemas for the construction of an "auxiliary econometric model" for the social security system by A. Coppini and G. Laina (June 1985).