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GRANULAR MATERIALS - THEIR APPLICATIONS AND PROBLEMS

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Specialists from a very wide variety of fields have been invited to today's symposium. Thus, mining, the iron and steel industry, the construction industry and soil mechanics, chemical engineering and also one of the most recent of sciences, reactor engineering, are all represented. Granular material stacks are the common subject. With such a varied audience it is naturally a requirement that, before the specialist papers proper begin, a general survey should be given of the meaning of the "granular material stack" as a special field, the disciplines and branches of industry in which they occur and what the common problems are. This task has fallen upon me. It ought really to have been the job of a specialist concerned with the granular materials field as a whole. Only part of this field lies in my own area of work so that I can give only a subjective treatment of the topic within the scope of this paper. My specialization is furnaces and it is therefore understandable that I shall choose for preference examples from this field.

Let me begin my talk with the simple statement that in the field of solid matter there are large groups of cohesive molecules referred to as grains, particles, lumps, etc., such as stones, grains of sands, lumps of coke, lumps of ore, or even clay. A number of such discrete pieces can be placed together and also on top of each other. If their number is sufficiently large they are called a stack, though other designations, if sometimes of a rather different significance, are also common. It must be left to the judgment of the observer to decide the number of discrete pieces above which the term stack should be used.

Regarding the external geometry of the stack, apart from the base area, it may be bounded laterally either by fluids only or by solid walls. In the first case, the description "open-air stacks" can be used, in the second the term "container stacks". The borderline between the two types is, however, vague since there are stacks which are only partially bounded laterally by solid walls.

As regards the position of the individual pieces within the stack, they need not necessarily retain their original location. They may, rather, be displaced either together or relative to each other under the influence of outside forces. A distinction must therefore be made between static and moving stacks. A stack is, however, composed not only of the discrete pieces but also of the interstices formed by the solid particles. The interstices are occupied as a rule by a gas or a liquid, or sometimes by both simultaneously. These gaseous and liquid substances may be at rest but they can also flow through the stack in any desired direction.

When a stack is made up of very different solid, liquid and gaseous substances simultaneously, interactions between the materials involved are observed, which are related to the wide field of thermal, material and impulse exchange. The stacked material, all the flow media present and the confining walls may play a part here in any combination.

The properties involved, both those of the discrete particles and of the complete stack, will now be discussed more fully together with the interactions between the partners named, with examples.

It will also be shown that, despite the multiplicity of external geometries and purposes involved, the problems raised are frequently the same or at least similar, although it cannot be claimed that the field of material stacks has been thoroughly investigated.

Column 1 in Table 1 gives a breakdown of the key-words pertaining to the properties of stack materials, a distinction being made between the material and geometric properties. The headings chemical and physical properties cover all the material properties which may be determining factors in any given stack. Fillers of the kind used in the chemical industry may be quoted as an example. The requirements often imposed here include resistance to acids, low water absorption and high abrasion resistance and breaking strength. The external geometry of the filler must also be such as to offer the largest possible effective surface area, the key-words "regular" and "irregular" in this table covering all conceivable shapes.

Fig. 1 shows some regular, discrete particles of various types of stacks. Fig 1a shows an Intalox saddle filler, 1b a Raschig ring, 1c a Pall ring, 1d a sphere and 1e a Berl saddle filler. These fillers are usually made of ceramic materials. They are employed mainly in the chemical industry to improve heat and material in distillery columns, scrubbers and coolers and also as catalysts in reactors, and distributing layers for gases and liquids.

Fig. 2 shows, in contrast, some irregular discrete bodies of bulk materials, as charged into blast furnace, i.e., some pieces of coke, pieces of Kiruna ore and some sinter fragments.

Column 2 in Table 1 lists the key-words pertaining to the stack structure. The composition of the stack is of interest in this connection, and here a distinction must be made between single-grain and multi-grain stacks; the arrangement of the individual fillers relative to each other, defined by the key-words ordered, random, homogeneous and non-homogeneous, is also important, as well as the boundary which determines whether the stack is of the open type or is confined.

Columns of fillers as used in the chemical industry can be adduced as an example of single grain type stacks, which thus consist of bodies of equal size. Multi-grain type stacks, which are a mixture of bodies of different sizes, include blast furnace, coke oven or sintering belt charges.

Fig. 3 shows as an example an idealized multi-grain stack of spheres of two diameters (Ref. 1), while effects of the composition of such multi-grain stacks on the void fraction (Ref. 1) are shown in Fig. 4. Here the ratio of the void fraction of the mixture ε_m to that of the single-grain-size bed ε_k is plotted versus the volumetric fraction of small spheres V_k/V or of large spheres V_g/V, with the ratio of their diameters d_K/d_g as a parameter; it can be observed that the void fraction in multi-grain-size beds is substantially lower than in single-grain stacks. The way in which the void fraction and hence the composition of the bed influences drag is shown elsewhere.

Fig. 5 illustrates the concept of "arrangement", an octahedral packing being shown on the left and a cubic packing on the right. Ordered beds are, however, comparatively rare in engineering. Random beds are much more frequent in engineering applications such as the bed of spheres shown in Fig. 6. The arrangement of the individual bodies is here entirely arbitrary and can at best be described by statistical laws.

Mention has already been made of the fact that in regard to boundaries a distinction must be drawn between open-air and container stacks.

Those of the first type mainly encompass the storage in a large space of bulk materials or in the open air, e.g., the storage of sand, gravel, crushed rock, etc., while container stacks are bounded by walls, as already mentioned. The significance of the wall as an element in the description of a stack can be seen from Fig. 7, which shows the void fraction of a bed of spheres in the vicinity of a container wall (Ref. 2). Immediately next to the wall the void fraction $\varepsilon = 1$; it then alternates between several minimum and maximum values which slowly decline. This phenomenon can be explained by the fact that the spheres right next to the wall are in a regular array, which gradually disappears towards the inside of the bed until, some distance from the wall, it becomes disordered. Both curves hold good for various ratios d/D, where d is the sphere diameter and D the container diameter.

By a combination of the key-words listed above, for the properties of the discrete bodies (Column 1) and the bed structure (Column 2), stacks encountered in engineering practice can be to a large extent described; they may, for example, be cubically arranged, single-grain-size beds of spherical bodies or perhaps random, multi-grain-size stacks of granular bodies in the open. Any other desired combinations are possible.

Columns 3 and 4 list the key-words that refer to the static and dynamic interactions mentioned at the beginning between the stack material, the various flow media and the container wall. Here a distinction is made, in Column 3, between stationary and moving beds, which may or may not have a fluid passing through them, while Column 4 covers thermal and material exchange.

As mentioned earlier, open, stationary stacks occur mainly in the storage of bulk materials such as coal, sand and ore. Characteristic features which either spring from or influence the said interactions are: the natural slope or angle of repose, the bulk density, water absorption capacity, spontaneous combustion of coal heaps, dust binding in open air stacks and many others. The disastrous results that can ensue if the characteristics quoted are disregarded or insufficiently known were shown only recently by the slag heap slide that occurred in Wales, causing serious material damage and many deaths.

Typical examples of enclosed, stationary stacks are bulk materials in bunkers and silos. A difficult problem here is the calculation of the pressure variation on the side walls and base, which is of great importance to the structural design of such installations. Fig. 8 shows a curve for the pressure versus the height of the bed in a silo, after Pieper (Ref. 3).

There are other problems, e.g., when bulk materials are ventilated, when a cover gas is used with materials which are explosive or subject to caking.

It has already been mentioned that stacks do not always remain stationary but in many of them the material is in motion. On closer investigation it is observed that in many cases this movement is due to gravity or other forces, as is also indicated in Table 1.

Movements of bulk material due to gravity arise where, for example, material is withdrawn from an enclosed stack, or where the stack changes owing to chemical or physical reactions in specific zones. Examples of this are the flow of material out of bunkers and silos, the discharge of fuel elements from the Jülich pebble-bed reactor and the settling of the bulk material in blast furnaces and in furnaces for the gasification of solid fuels. An instance of such movement due to other forces is the rotary kiln, or the movement of stacks as a result of a flow impulse.

The settling of the bulk material in a bunker is shown in Fig. 9, after Kvapil (Ref. 4). It can be clearly seen that the middle zone settles first and that it is only then that the material against the side walls subsides. Even such a simple process as the flowing of bulk material from a bunker poses numerous problems. Separation phenomena occur, for example, conditioned mainly by the varying density, the grain size, the grain shape, the friction, the water content and the container geometry. Similar separation phenomena to those occurring during the flowing of bulk material out of a container also arise, however, in the charging of furnace and, for example, during the heaping of bulk materials. The movement in a rotary kiln is shown schematically in Fig. 10, taken from Taubmann (Ref. 5). It can be seen that the material is first entrained in the sense of the peripheral motion of the kiln shell owing to static friction and subsequently falls back again through the effect of gravity.

Another common phenomenon observed in material movements in stacks is arching, meaning the formation of humps in the stack so that the whole stack or parts of it are suspended. Attempts are frequently made to prevent arching by installing means of aiding discharge and thereby to improve the movements of the bulk materials.

Fig. 11 shows such arching in a bed of fire clay fragments bounded by two parallel glass sheets. The parameters influencing arching are the same as the factors already quoted in discussing separation, as well as the material properties which promote caking. A special case, however, of desired arching is the formation of combustion spaces in front of tuyères in blast furnaces, as illustrated schematically in Fig. 12 (Ref. 7). It has not proved possible so far to clarify this process fully. The main factors involved here are pressure forces in the bed, internal friction, grain shape, grain size, the material density and above all, the impulse of the incoming gas jet. Other questions relating to the movement of bulk materials under gravity are the separation of the material, the velocity distribution and the tracking of the path of the individual particles.

Apart from the movement of the bulk material itself, however, a flow of gases or liquids in the voids in the bulk material has to be considered in many cases, a distinction being made between single- and multi-phase flow, as indicated in Table 1. The flow in a stack is single-phase if, for example, a specific gas flows through a bed in order to heat it or be heated by it. If on the other hand, a liquid flows downwards through the bed and a gas flows in the opposite direction, then the flow is multi-phase. Problems which arise here include the calculation of the pressure drop, the determination of the velocity distribution and axial mixing in the flow. Fig. 13 shows, as an example of the above correlations, the pressure drop coefficient ψ versus the Reynolds number Re for beds of equal-sized spheres.

Similar correlations exist, however, in respect of beds of granular material and for multi-grain-size beds. How the coefficients are defined has already been discussed elsewhere (Ref. 1). If the fluid exceeds a specific limit velocity, the solid bed expands and the complete system undergoes a qualitative change. In the case of multi-grain-size beds this can cause the ejection of smaller particles from the bed.

The laws governing multi-phase flow through beds are considerably more complicated, e.g., when a gas flows in the opposite direction to a downwards trickling liquid. In this case the water content of the material column, at a constant rate of liquid feed, increases for rising gas velocity up to a point where the whole bed is flooded.The increase in the water content of the bed is, however, also accompagnied by a marked pressure rise in the column, as evident from Fig. 14 (Ref. 8). Here the pressure drop per unit length in a coke bed is plotted versus the gas velocity in the column (assumed to be empty), with the rate of trickle flow as a parameter. The gas used in this case was air and the liquid water. The bottom curve applies to a dry bed, i.e., one through which only air passes. The point at which flooding was observed is also marked. Apart from these interrelations, however, the distribution of the individual flows over the cross-section is also important in beds with a multi-phase flow through them and in this connection mention should be made of the usually undesirable ease of flow in the peripheral zone. An example of a bed with a two-phase flow through it is shown in Fig. 15, after Kiesskalt (Ref. 9), with the flow density distribution at various heights also indicated on the right.

Finally, the key-words listed in the last column of Table 1 describe the exchange phenomena occurring in beds. Only thermal and material exchanges are mentioned, however, the wide field of impulse exchange being already catered for implicitly in Column 3 in the treatment of flow mechanisms in beds. The bed material, all the flow media and the boundary walls may, as already mentioned, play a part in the exchange phenomena dealt with in Column 4. In exchange in which the bed material itself plays a part, transfer phenomena must usually be considered also within the discrete pieces of the stack. Whether the bed is static or mobile is also important and, lastly, exchange phenomena occur perpendicular to and along the direction of flow if there are temperature and concentration differences in it.

The purpose of these exchanges is either a desired physical or chemical change in the bed material itself or in the fluids passing through it. The processes are purely physical when, for example, temperature changes or, in other cases, simultaneous concentration changes in the substances involved are aimed at, e.g., in drying. In the case of chemical reactions in beds, on the other hand, oxidation, reduction or catalytic conversion may take place and, finally, mention must be made of nuclear reactions. Both subdivisions of Column 4 indicate the manner in which heat and matter are transferred in beds. Thus heat transfer can take place by conduction, by convection or by radiation. In material exchange the analogy with radiation is of no interest here, so that only material transport by conduction and by convection is quoted. It may also be mentioned that in many cases thermal and material exchange occur simultaneously and that both processes may be physically and hence mathematically connected. It is then important to the study of these processes that conduction for thermal and material exchange and convection for both types of exchange be capable for formal description by the same differential equations, so that if one process is known the other can be understood. As is known, there is said to be an analogy between thermal and material exchange, but there are also defects in the analogy.

Examples of beds in which combinations of the reactions mentioned occur, according to a systematic study by Luther (Ref. 10), are rectifying columns, absorption towers and scrubbers, fixed-bed reactors and movable bed reactors, driers and sintering bands, and, lastly, the pebble-bed reactor here in Jülich. It can be shown by analyses of the mechanisms at work in these processes that in many cases distinct exchange processes can be described by the same mathematical equations. Purely theoretical solutions are, however, possible in only a few cases so that these processes are generally investigated experimentally. The dimensionless representation of the individual parameters has been proved sound for this, and is shown as an example in Fig. 16 in respect of heat transfer between gases and the bodies composing beds of spheres. Here the dimensionless Nusselt number Nu (heat transfer coefficient) is plotted over the Reynolds number Re, these coefficients having already been defined elsewhere (Ref. 11). The points marked apply both to single-grain and multi-grain-size beds. Analogous representations are known for other exchange phenomena in beds.

These remarks should close with a discussion of some further examples from the broad field of beds. Thus, Fig. 17 shows a pebble heater for preheating air or other gases to high temperatures (Ref. 12). In this process a pebble bed is constantly circulated and preheated in the upper section of the device to the desired temperature by a counter flow of hot gas, the heat absorbed in the top section then being transferred to the cold air at the bottom. The cold spheres are then discharged at the bottom and fed back in at the top so that the process begins again. This is then a moving bed with a simultaneous flow of gas. The only exchange phenomena to be considered are heat transfer by convection and conduction in the gas and the solid phase. In order to calculate the heat transfer, however, the flow distribution over the cross-section must be known. Sand-bed reactors for cracking petroleum products and for the production of coke briquettes, on which Luther (Ref. 10) reported only recently, work on much the same principle, although the heat transfer conditions are considerably more complicated and chemical conversions also occur.

As a further example Fig. 18 shows a modern kiln for the burning of limestone, in which gas is fed in and removed at a very wide range of points (Ref. 13). Air is injected at the bottom to cool the calcined limestone and then to sustain combustion. The gaseous or liquid fuel is fed into the kiln at about the middle through the twin row of burners as shown. The exhausted gases are finally extracted upwards after surrendering most of their enthalpy to the limestone, which passes downwards through the kiln, i.e., in the opposite direction. The laws for calculating the pressure drops must be known for designing the various blowers if the individual gas streams are actually to flow in the desired direction. The motion of the solid material is comparable to that in the pebble heater illustrated previously, since material is discharged from the bottom of the kiln. However, in contrast with the processes in the pebble heater, the lumps of material are partially broken down owing to the desired lime decomposition, so that it is very difficult to have complete control over the flow phenomena and hence the heat and material exchange, too, in the case of the lime kiln. Furthermore, arching due to caking can interfere seriously with the settling of the charge.

Finally, Fig. 19 shows a blast furnace in which the flow phenomena in charge and the heat and material exchange are even more complicated than in the lime kiln quoted above (Ref. 14). While the flow through the part of the charge occupying the top of the furnace is single-phase, the slag and the raw iron in the bottom must also be considered since they flow in the opposite direction to the gas. Allowance must also be made for the fact that the charge at the top of the furnace contains coke and ore, while there is only coke at the bottom. Furthermore, plastic states occur in an intermediate layer, causing the flow phenomena to become highly unpredictable. Finally, mention should be made of the fact that, in contrast with the examples shown previously, only liquids are discharged at the base of the furnace and not solid materials. The settling of the charge is, therefore, caused here partly by chemical and partly by physical changes.

One might be tempted, then, to consider stacks of bulk materials in technical applications as a modern achievement, but a glance back over history proves that they have been known to and used by mankind since the earliest times. Thus Fig. 20 shows on the left the remains and on the right a reconstruction of an early medieval furnace which was found in the Altena district and belonged to a bloomery iron forge (Ref. 15). At that time iron was smelted from beds of ore and charcoal. The charging of a somewhat later furnace is shown in Fig. 21, being based on a well known illustration by Agricola (Ref. 16) dating from 1556. Comparison with the human figures in the picture shows, too, that these smelting hearths were fairly small in those days. They are somewhat reminiscent of the way in which iron was smelted in China until very recently.

This brief look at historical furnaces leads us immediately to the most modern example of the application of a stack, namely, the pebble-bed reactor, which will be fully discussed in the next paper.

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Table 1 Characteristics of beds of granular material

1 Properties of the discrete bodies	2 Bed structure	3 Behaviour of static and moving beds	Heat and matter transfer in beds*
Material properties	Composition	Static beds	Heat transfer
chemical	single-size		
physical	multi-size	Moving beds/stacks	due to conduction
		gravity induced movement	due to convection due to radiation
Geometry	Arrangement	movement due to other forces	Material transfer
regular	ordered		due to diffusion
irregular	homogeneous		due to convection
	non-homogeneous	Flows in beds/stacks	
	disordered	single-phase	
	homogeneous	multi-phase	
	non-homogeneous		
	Boundary		*with or without chemical or nuclear reactions
	open		
	contained		

JENIKE'S METHOD FOR THE DESIGN OF BINS AND HOPPERS

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Abstract

An outline is given of a theory developed by Jenike for designing bins and hoppers for frictional-cohesive solids. Following a representation of the flow limits and strength of bulk materials, the stress conditions in bins and hoppers and the establishment of flow criteria, a demonstration is given by way of example of how, using Jenike's method, bins and hoppers can be designed so that their contents flow smoothly and arching is prevented.

1. Introduction

The storage and flow properties of bulk materials are assuming increasing importance in material and energy conversion processes. A bunker is required to discharge material smoothly and in precise quantities at predetermined times. The well-known phenomena of arching, pipe formation and over-rapid outflow of material which make discharge irregular or cause it to cease completely are functioning defects which can be prevented by appropriate design of the bunker.

Jenike and his co-workers, using the methods of continuum mechanics, plasticity theory and soil mechanics, have established a closed theory for the gravity flow of frictional-cohesive bulk solids. Jenike also developed a test method for the laboratory determination of those physical constants most important to the design of bunkers.

The results of his work are contained in a series of articles (Refs. 1-7). While Jenike's approach is used successfully in the English-speaking countries, it still seems to be largely unknown on the continent of Europe. The purpose of this paper is to outline Jenike's theory with reference to arching.

Fig. 1 shows a bunker filled to a level H with a bulk solid. It consists of a vertical section joined at a height h' to a funnel-shaped discharge section with its apex at a height h = 0. Alongside it are three pressure versus height curves, their significance being as follows:

1) The quantity σ_1 is the largest main stress acting on a flowing element at the height considered. This stress increases with the depth as far as the transition to the convergent section, and then decreases again, reaching zero at the apex. In the case of silos the point of maximum stress is in practice reached earlier and would then remain constant as far as the height h'. A linear stress curve is represented in the convergent section. It will be shown later that this holds at least near the apex.

2) The quantity f_c represents the strength of the material with any desired orientation of the material surface. This quantity is illustrated by the theoretical experiment shown in Fig. 2. A specimen of a bulk solid is homogenously compressed by a stress σ_1 in a cylindrical container. If the container is removed and the material subjected to a compressive stress acting in the same direction as σ_1, a pressure f_c is needed to crush the column of material, this pressure representing the strength of the material. It is evident, and universally known from experiments with sand boxes, that the strength f_c increases with the compacting stress. This gives the stress curve for f_c shown

schematically in Fig. 1b. It is worth noting that f_c does not vanish at the levels 0 and H. The reason is that even in the non-compressed condition cohesive material has a certain strength due to adhesion. This is almost true of fine-grained solids with particle sizes $< 100\ \mu m$.

3) The quantity σ_1' represents the stress that must exist at the bearing surfaces of a stable arch. As is also shown, it is proportional to the width or diameter of the bunker at the point considered. This gives the straight curve in Fig. 1b.

If the pressure curves are known, the following flow criterion can be established for arching: if the bearing surface stress σ_1' exceeds the trength f_c of the material, then a stable arch is not possible, i.e., the strength of the material is not sufficient for arching to occur. In the case considered in Fig. 1 the discharge orifice of the bunker must, therefore, be positioned no lower than the level h^*, precisely where $f_c = \sigma_1'$. How, then, are the three stress quantities and their variation with height established?

2. Bearing surface stress σ_1'

The derivation of the bearing surface stress for a stable arch σ_1' is illustrated by a simplified representation (Fig.3). The following assumptions were made:

1) the arch is parabolic;

2) the material above exerts no pressure on the arch;

3) the arch has the vertical unit thickness 1 so that the stress q due to gravity is uniformly distributed over the width b;

4) the discharge funnel is wedge-shaped with slope Θ';

5) no support is offered by the front walls.

If the angle of friction between the wall and the material is ϕ_x, the maximum slope of the arch at the bearing surface will be A $\delta = \Theta' + \phi_x$. An equilibrium analysis gives, for this case, the following equation for the bearing surface stress σ_1':

$$\sigma_1' = \frac{b \cdot g \cdot \rho_s}{\sin 2\delta} \qquad (1)$$

in which g is the acceleration due to gravity and ρ_s the bulk density. In the case of a circular discharge funnel there would also be a factor of 2 in the denominator. In addition to this simplified calculation Jenike also made a more precise one (Ref. 8), in which allowance is made for the weight of overlying layers. Like Equation 1, the more precise calculation shows a linear relationship between the bearing stress σ_1' and the width or diameter of the bunker.

Strength f_c

The strength of the material is determined experimentally. The theoretical experiment in Fig. 2, in which the material is compressed in a mould which is then removed before crushing the column of particles, is not suitable for the purpose. For one thing, it is difficult to achieve homogeneous compaction in a cylinder and, furthermore, the cylinder wall would have to be frictionless for compaction to be unaffected by wall friction and for it to be possible to remove the cylinder from around the material without disturbing it.

Thus Jenike developed a shearing device derived from soil mechanics practice for determining shear curves, from which the quantity f_c can be established graphically. A curve of this type is shown in Fig. 4. The stress needed to shear a sample subjected to the normal stress σ is plotted against the latter. As in soil mechanics, compressive stresses and contractions are positive quantities. A number of samples are brought to the same, specified state of compaction and then sheared under various compressive stresses σ. The curve thus obtained is termed by Jenike the "yield locus".

The yield locus is a yield point in the meaning used in plasticity theory and represents the stress conditions that must be exerted on a material element for it to deform plastically. A characteristic quantity of the yield locus is the large principal stress σ_1 of the Mohr's circle of stress which is tangential to it at the terminal point e. If a circle of stress is now drawn through the origin of the coordinates tangential to the yield locus, the strength f_c of the material compacted by the principal stress σ_1 is obtained. The quantity f_c represents the large principal stress of the stress condition in which the second principal stress is equal to zero. This is precisely the stress condition prevailing in a bulk material arch. A linearized yield locus has also been drawn in Fig. 4, tangential to the two circles of stress plotted and forming an included angle ϕ with the σ axis. The linearized yield locus is necessary for further mathematical treatment. The ratio of σ_1 to f_c characterizes the ability of the material to flow. It is called, after Jenike, the flow function ff_c.

$$ff_c = \frac{\sigma_1}{f_c} \qquad (2)$$

Every compaction condition characterized by the principal stress σ_1 has, therefore, a corresponding yield locus (Fig. 5). If the envelope of the largest Mohr's circles of stress is drawn for individual yield loci, an approximately straight line is obtained passing through the origin of the coordinates and inclined at an angle ϕ_e to the σ axis.

This straight line is called the effective yield locus, after Jenike, and characterizes the condition of steady flow. The meaning of this can only be briefly indicated.

A material element to which the yield locus 1 (Fig. 5) corresponds is stressed in such a way that the corresponding Mohr's circle is tangential to the yield locus at point A. According to the normality principle, based on the plastic potential after v. Mises (Ref. 9), the elongation rate vector, which determines the change of volume in incipient plastic flow, is perpendicular to the flow plane. If isotropic conditions are assumed, i.e., the directions of the principal stresses and the principal elongation rates are assumed to coincide, a formal elongation rate vector $\bar{\dot{\varepsilon}}$ can be drawn perpendicular to the yield locus.

In isotropic conditions the coordinates σ and τ also represent the axes of the normal elongation rate $\dot{\varepsilon}$ and the rate of change in slope $\dot{\gamma}/2$. As can be seen, the vector $\bar{\dot{\varepsilon}}$ has a negative component. The material will therefore be elongated. However, this gives it a lower degree of compaction with a correspondingly lower yield locus. This extension ceases when the terminal point of a yield is reached at which, as Jenike was able to show, the vector $\bar{\dot{\varepsilon}}$ is vertical to the $\dot{\varepsilon}$ axis and thus causes no further change in volume. This is just what happens in steady flow.

Stress variation σ_1

Jenike calculated the stress variation for the steady-flow condition, such as prevails, say, in a rectangular bunker with a wedge-shaped discharge funnel and for the axisymmetric flow condition in a circular bunker. Three stresses are of prime importance to the steady flow condition (Fig. 6): the normal stresses σ_x and σ_y and the shear stress τ_{xy}. In the axisymmetric case, which is not to be dealt with here, a circumferential stress σ_α also occurs. The plane xy is a plane through the bunker axis, the x axis lies in the same direction as the axis and the y axis is perpendicular to it. There are therefore three unknowns to be determined.

a) for the steady-flow case

and b) for the incipient-flow case.

The following equations can be written for them:

Equilibrium conditions in the directions x and y are covered by

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = g \cdot \rho_s \qquad (3)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \qquad (4)$$

For steady flow the following relationship applies as an equation for the effective yield locus (Fig. 5):

$$(\sigma_x + \sigma_y) \cdot \sin \phi_e - {(\sigma_x - \sigma_y)^2 + 4 \cdot \tau_{xy}^2}^{1/2} = 0 \qquad (5)$$

and for the incipient flow the following relationship applies as an equation for the linearized yield locus (broken line in Fig. 4):

$$(\sigma_x + \sigma_y) \cdot \sin \phi - {(\sigma_x - \sigma_y)^2 + 4 \cdot \tau_{xy}^2}^{1/2}$$

$$+ f_c\,(1 - \sin \phi) = 0 \qquad (5b)$$

The fourth equation is another relationship between the largest principal stress σ_1 and the bulk density ρ_s:

$$\rho_s = \rho_s(\sigma_1) \tag{6}$$

If the individual stresses are replaced by a mean stress σ and the included angle ω between the direction of the principal stress σ_1 and the x axis (Fig. 6), we obtain a system of quasi-linear partial differential equations. If allowance is made for the peripheral conditions, the set of equations can be solved numerically by means of the characteristics method. The field can thus be overlaid with a characteristics grid. Dr Cutress, who demonstrated the evolution of the lines of slip experimentally with X-ray images, will report on this further in a later paper.

Since the difficulties with bunker storage arise mainly in the vicinity of the discharge orifice, a knowledge of the stresses close to it is sufficient for establishing flow criteria. They are independent of peripherical conditions àt the top of the bed and are affected only by conditions at the funnel walls. If polar coordinates are introduced (Fig. 7), the following approximate equation can be written:

$$\sigma_1 = r \cdot g \cdot \rho_s(r, \Theta) \cdot s(r, \Theta) \tag{7}$$

in which r denotes the radius vector from the base apex to a material element and Θ the included angle between the radius vector and the funnel axis. The function $s(r,\Theta)$ is derived from the numerical solution of the set equations. As was proved by Johanson (Ref. 10), a collaborator of Jenike, the stress field near the funnel apex and thus, with sufficient accuracy near the discharge orifice, approximates to a specific stress field referred to as the "radial stress field", in respect of which the bulk density ρ_s is independent of r and Θ and the function s is independent of r. Equation 7 is thus simplified, giving Equation 8:

$$\sigma_1 = r \cdot g \cdot \rho_s \cdot s(\Theta) \qquad (8)$$

The radial stress field requires the solution of only two ordinary differential equations and is therefore considerably easier to determine. The function $s(\Theta)$ depends not only on Θ but also on the slope of the funnel wall Θ', the friction angle ϕ_x between the material and the wall and the angle ϕ_e of the effective yield locus (Fig. 5). Jenike reproduces the function s in the form of graphs (Ref. 2).

It follows from Equation 8 that σ_1 is linearly dependent on r and thus on the funnel width or diameter. For $r = 0$, σ_1 vanishes and so does the bearing surface stress σ_1' of an arch. If the ratio of σ_1 to σ_1' is established, it is seen to be constant for a particular combination of bulk material and discharge funnel. Jenike terms it the flow factor ff:

$$ff = \frac{\sigma_1}{\sigma_1'} \qquad (9)$$

Jenike gives graphs of the flow factor values (Refs. 2 and 7).

5. Prevention of arching

It will now be shown, using Fig. 8 as a reference, how, knowing the flow factor ff and the flow function ff_c, the minimum dimensions of the discharges orifice are determined so as to rule out arching. In Fig. 8 the bearing surfaces stress σ_1' of an arch and the strength f_c of the material are entered on the ordinate and the stress σ_1 on the abscissa. The flow factor ff takes the form of a straight line through the origin of the coordinates while the experimentally determined flow function is a curved line.

$$ff = \frac{\sigma_1}{\sigma_1'} \quad \text{const.} \tag{9}$$

$$ff = \frac{\sigma_1}{f_c} \quad \text{const.} \tag{2}$$

Hence it follows from the flow criterion established initially, which stated that

$$\sigma_1' \geq f_c \tag{10}$$

is necessary to prevent arching, that the flow function ff_c must be less than the flow factor ff, i.e.,

$$ff_c \geq ff \tag{11}$$

The point of intersection A of the two curves determines a value of $f_{cA} = \sigma_1{}'_A$ and thus a minimum discharge orifice size.

A similar study was performed by Jenike to prevent pipe formation (Refs. 2 and 11). It resulted in a flow factor for pipe formation which can be plotted in Fig. 8. Its point of intersection with the flow function then determines a minimum orifice size which rules out pipe formation.

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SOME PARAMETERS CHARACTERIZING THE FLOW BEHAVIOUR OF SPHERICAL AND NON-SPHERICAL BULK MATERIALS

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Abstract

Experiments on the discharge of bulk materials from bunkers are reported on, with real bulk materials such as alluvial stones, quartz fragments, granite and greywacke compared with spheres, representing the ideal bulk material. The varying weight flows are described by dimensionless discharge-reducing factors. The influence of various parameters on the discharge-reducing factor is investigated.

1. General

In my short paper and the following by Mr. Matthé an account is given of investigations carried out at the Mining Institute of the Technische Hochschule Clausthal (Refs. 1-12). While our work was originally tailored to the requirements of the mining industry, in which the use of bunkers for storage purposes at many stages of operations is well known, it soon became clear that our researches are also of great relevance to other industries. We became particularly aware of how topical these matters are in 1966 at a Symposium held at our Institute and entitled "The behaviour of bulk materials in bunkers and silos", when we were given valuable stimuli for further work.

2. Flow behaviour of bulk materials

So far we have been shown clearly how a bulk material moves on being discharged from a bunker or silo. We know that the type of movement is influenced by factors which are themselves conditioned by the dimensioning and nature of the bunker and by the bulk material itself. The extent to which the nature of the material affects the discharge process is shown by the following test results. The experiments were carried out in cylindrical model bunkers using the following bulk materials: glass spheres, pebbles, quartz fragments, granite chippings and greywacke chippings, thus covering a range of grain shapes from round to disc-shaped. Grain sizes and container dimensions were matched to each other in such a way that the results produced clear conclusions.

2.1 Influence of the ratio of discharge orifice to grain diameter

Fig 1 shows a comparison of the discharge rate M of various bulk materials for grain diameter ranges of 4.0 - 2.5 mm plotted versus the ratio of discharge orifice diameter b to grain diameter d. The container floor is horizontal, the discharge orifices circular and central. In evaluating the test results graphically it was found that the discharge rate rises parabolically with increasing V. The more the shape of the individual components of the bulk material deviates from the ideal spherical configuration, the flatter the discharge rate curve becomes. This means, however, that a spherical bulk material can be discharged at a higher rate than, for examples, pebbles or greywacke chips if the ratio of discharge orifice diameter to grain diameter is equally large. It is also evident that the discharge rate falls for decreasing grain size, for a constant ratio of orifice diameter to grain diameter.

The influence of the ratio of container base to orifice area can also be seen from the curve for these materials (Fig. 2). This ratio is plotted on the upper abscissa as K. The values of K take the form of straight lines joining the points on the discharge rate parabolae at which the ratio of container base area to discharge orifice area is the same. The curve for glass spheres is on the left in the figure and that for greywacke chippings on the right. The straight lines have an approximately logarithmic spacing and become steeper with increasing deviation from the spherical shape. It can be seen from these curves that the more the bulk material deviates from the spherical shape, the smaller is the range covered by the discharge rate as a function of the grain size.

Although the lower part of the curves is built up from the general pattern (broken lines), it is striking that all the curves originate from the point D = 3. This corresponds to the ratio defined by Takahashi and implies that the diameter of a discharge orifice must theoretically be at least three times as great as the grain diameter, with this statement increasingly applicable to spherical and equal-sized bulk material components.

2.2 Influence of bunker base slope

In order to permit comparison of the discharge rate of various bulk materials with specific grain diameters, as a function of the slope α of the bottom of the bunker, the experiments were evaluated as shown in Fig. 3. In the cylindrical model bunker, with the orifices of the various funnel-shaped discharge sections on the bunker axis, the ratio of bunker base area to orifice area was constant. The experiments were carried out using the following materials: glass spheres, pebbles, quartz fragments, granite chippings and greywacke chips.

It can be seen from Fig. 3 that for decreasing grain size the discharge per unit time increases more quickly the closer the particles approximate to the ideal spherical shape. The ratio between M and the floor slope α shows that there is an almost parabolic dependence, i.e., the quantity discharged rises with increasing α. The curves' gradients increase from about the point at which α exceeds the internal friction in the bulk material. As a result, M increases in glass spheres at a lower α than in bulk materials whose internal friction is greater owing to the deviation from the spherical shape.

It is also striking here that varying grain size exerts an influence only in ideal bulk materials (glass spheres), particularly with regard to an increasing discharge rate. The extremely small influence is especially noticeable with greywacke chips, which showed the worst flow properties of all the materials investigated.

It is, however, all the more remarkable that there exist such big differences between the graph for glass spheres and the one for rounded pebbles. In contrast, the graphs for pebbles, quartz fragments, granite and greywacke fall into a narrow range, although it might be assumed, in view of the grain shape, that pebbles should resemble glass spheres more closely than greywacke chips as regards their flow characteristics. In this case, however, the predominant factor is internal friction which is due not only to grain size but also to surface roughness, amongst other things.

Let me say a brief word about the use of special shapes of bunker discharge outlet. As can be appreciated, the nozzle type is suited to discharge conditions when the funnel angle is 75°. This fact is important because a nozzle-shaped discharge arrangement requires less material and also has a lower installed height.

In order to establish the influence of some of the bulk material factors and the base angle of a bunker on the discharge per unit time, the characteristic values of the bulk materials investigated were plotted as parameters. The same bunker dimensions and grain sizes were assumed.

The direct relationship between the internal friction μ of the bulk materials investigated and the quantity discharged is illustrated by Fig. 4. Discharge increases as internal friction declines, the rate rising for decreasing μ. The same tendency is evident in regard to the void fraction and, correspondingly the void volume.

The influence of the container floor slope α shows that the quantity discharged per unit time increases approximately logarithmically with rising α. At the same time the difference between the smallest and largest floor angles (0 and 75° in this case) diminishes for increasing internal friction. This means that the difference decreases the more the bulk material deviates from the spherical shape, a phenomenon also to be seen from the graphs already shown.

With decreasing sliding resistance, i.e., the friction between the bulk material and the container walls, the quantity discharged per unit time also increases. The floor angle has a similar influence, but the curves bend in the opposite direction to those of the internal friction μ.

The influence of the bulk density γ_e is shown in Fig. 5 for identical experimental conditions. Here the quantity discharged increases with rising bulk density, a phenomenon which contrasts with the other bulk material factors. This also applies to the dependence on the floor angle α. It will be seen that, in comparison with bulk material deviating most from the spherical shape (in the case greywacke), it is the bed of spheres having the highest density which displays the highest discharge rate.

2.3 Discharge-reducing factor

It has been established from the experiments carried out so far that bulk materials with non-uniform particle shape display other discharge phenomena. The discharge-reducing factor A was developed to characterize this diversity. It expresses in the form of a dimensionless factor the number of times the quantity discharged per unit time for an ideal material (glass spheres) must be multiplied in order to obtain the discharge rate for a specific material. For a material with identical grain diameter this factor remains constant in varying discharge conditions (Fig. 6).

For quantities discharged per unit time M from a container with a flat floor and a constant ratio of container base area to orifice cross-section, discharge-reducing factors A were developed for pebbles, quartz fragments, granite and greywacke at grain sizes of 4.0 and 2.5 mm using the following equation:

$$A = \frac{M_{\text{bulk material}}}{M_{\text{glass spheres}}}$$

Fig. 6 shows the dependence of the quantity discharged per unit time M on the flow-reducing factor A, with A = 1 in the case of glass spheres. It can also be seen from it that there is a large difference between the discharge rate of glass spheres and rounded pebbles, larger than that between pebbles and greywacke chips, the latter being defined as sharp-edged. With decreasing grain diameter the discharge-reducing factor A diminishes with rising discharge rate.

In this connection it is interesting to investigate the influence of the internal friction μ, the void fraction ε and the bulk density γ_e on the discharge-reducing factor A (Fig. 7). The top graph shows that the factor A falls as the internal friction μ rises, i.e., the quantity discharged per unit time declines. The curve for the void fraction is similar, as the middle graph shows. However, the values for grain sizes of 4.0 and 2.5 mm display so much scatter that only one zone of influence can be discerned. In the case of bulk density γ_e (bottom graph) the curves are approximately symmetrical with respect to each other, in line with the observation already made to the effect that with increasing γ_e the quantity discharged per unit time rises in accordance with the factor A.

The inclusion of further components determined by the material and the container is being handled by our Institute with the main aim of deriving values which can be transferred to practical applications.

2.4 Prerequisites for smooth discharge

From the studies of the quantity discharged per unit time as a function of the ratio container floor area/discharge orifice cross-section (K), the ranges in which smooth discharge of the material without obstructions or arching is ensured were established in Fig. 8. Dependence on K and D (the ratio of orifice diameter to grain diameter) is plotted. The shape of the limit curve can be transposed to larger bunkers and other types of bulk material with similar characteristics to the materials studied, since this graph shows dimensionless ratio numbers which combine dependence on bulk material and container dimensions. The curve indicates that for large K a small D is required (glass spheres). However, with a material having, for example, the characteristics of granite chippings, K is smaller and D larger, i.e., large cross-sections are required for a relatively small-grained material in order to obtain smooth discharge without arching.

In Fig. 9 we plotted the frequency of arching on a percentage basis, using the experimental results for the quantity discharged per unit time with varying floor angle and the same orifice and grain diameters. It shows that owing to their shape granite chippings most frequently tend to arch, but this is reduced with increasing conicity of the container bottom. It is therefore recommended that bunkers should be built with a floor angle that suits the properties of the material. Thus, for example, an inclination of about 50° is enough to keep the frequency of arching in granite chippings below 40%.

3. Pressure conditions

A brief reference should be made to how spherical and non-spherical bulk material react to pressure conditions in a bunker. In this connection let us examine the pressures exerted on a horizontal bunker floor (Fig. 10).

The influence of the properties of various bulk materials, which I have already discussed, affecting flow behaviour can also be extended, in a certain respect, to the pressure conditions. The left-hand half of the figure shows that the floor pressure rises with increasing height h or with increasing weight of the column of material, which is in line with the well-known findings of Janssen. It can be seen that, of all the bulk materials studied, glass spheres cause the highest and greywacke chips the lowest floor pressures. In other words, the curves become steeper with increasing angularity of the material. This shows that pressure absorption takes place in the column of material which becomes more pronounced with increasing deviation of the material from the spherical shape and with increasing weight or height of the column of material.

4. Conclusion

The aim of this paper has been to discuss some of the parameters characterizing the flow behaviour of spherical bulk materials. Although it is extremely difficult to determine fully all the quantitative factors involved, further investigations are in progress at the Mining Institute of the Technische Hochschule Clausthal, of which the next paper gives an example.

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Fig. 1. Discharge rate M for various bulk materials as a function of the ratio of orifice diameter d to grain diameter b

Fig. 2. Discharged rate M as a function of the ratio of container base area to discharge orifice area (K)

Fig. 3. Discharge rate M for various bulk materials as a function of bunker floor angle α.

Fig. 4. Influence of the internal friction μ on the discharge rate M with various bunker floor angles α.

Fig. 5. Influence of the bulk density γ_e on the discharge rate M with various bunker floor angles α.

Fig. 6. Interrelationship between the discharge rate M and the discharge-reducing factor for bulk materials with various grain diameters

Fig. 7. Influence of the internal friction μ (top), the void fraction ε (middle) and the bulk density γ_e (bottom) on the discharge-reducing factor A

Fig. 8. Limit values for smooth discharge of various bulk materials of the same grain size

Fig. 9. Influence of the floor angle α on the frequency of arching (%) with the same orifice and grain diameters

Fig. 10. Specific floor pressure P_{spec} (left) and pressure absorption ΔP (right) for the same grain diameter

THEORETICAL AND EXPERIMENTAL INVESTIGATION ON SEGREGATION EFFECTS IN STORED MATERIALS

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Abstract

Using dimensional analysis an attempt is made to define dimensionless characteristics affecting the segregation during the transposition of bulk solids. There follows a report on tests with a heap of spheres composed of two fractions of different diameters. The observed segregation is described by a segregation factor.

1. Segregation as part of the complex behaviour of bulk solids when flowing out of bunkers

Many "dependent variables" are involved in the storage of bulk solids in containers of any kind; in order to give a correct description of the operation of a bunker, it is necessary to identify them exactly, to know how they interact during the complex phenomenon of "bulk solid behaviour" and to arrive at constructive solutions for their interrelationships.

The phenomenon of segregation is one particular problem among others, such as the right choice of bunker size and shape in relation to the bulk solid to be stored, the charging and emptying systems, their shape and position in the bunker cross-section, the calculation of the pressure conditions in the bunker, the actual flow behaviour of the bulk materials, their tendency to arch and their intrinsic properties.

What is segregation? The following definition seems apt (Ref. 1): "If a bulk solid consisting of components of different shapes is moved, a segregation can be observed. This means that as a result of the moving of the bulk solids similar substances detach themselves from the mixture and are deposited separately".

Two kinds of segregation can be distinguished:

according to granule size and
according to specific weight.

In segregation according to granule size, which can be observed in mountain screes or coal slag heaps, the bigger lumps are separated out from the main mass towards the edge during the heaping process. The smaller granules collect in the centre of the heap.

Segregation according to the specific weight takes the form of a separation of bulk materials of two different specific weights. During the heaping operation the material with the lower weight tends to migrate outwards, the heavier material remaining in the centre of the column.

1.1 Theoretical considerations on sub-processes involved in segregation by granule size

In attempting to provide a quantitative description of the segregation process in bunkered heaps of spheres, the Mining Institute of Technische Hochschule Clausthal has hitherto confined its attention to segregation according to granule size. Here it is easy to give a mathematical description of segregation in the case of a free vertical fall. In the experiment, however, the segregation is not very marked, and becomes significant only when a horizontal component is added to the gravity acceleration. Jenike (Ref. 2) has derived the segregation process for this case from simple mathematical formulas.

Segregation by granule size on a cone of bulk material is particularly interesting since it is not immediately obvious why large particles are deposited at the edge, while particles of high specific gravity remain in the centre. The observations to be found in the literature do not always appear logically sound.

In order to form a better picture of the movement processes along such a cone, we will first follow in a simplified form the movement of individual particles of the heap on an inclined plane. With the aid of the equations of the laws of friction and conservation of energy it can be calculated that in the case of pure sliding friction large and small granules cover the same horizontal distance, since the path is calculated as independent of the size and mass of the granules. In the case of rolling friction on the other hand the path depends upon the diameter of the particles. Large granules are then able to cover a greater horizontal distance than small granules, if the heap has at least a partly rolling movement.

If we now define the segregation of a bulk material when poured into a cone as a purely surface phenomenon, so that granules at a certain distance from the cone surface are not displaced and take no part in the segregation, and if the cone surface is assumed to be a rough plane, the projections on which are smaller than the diameter of the largest granule, then a particle can overcome the obstacle of another particle only if a "positive" factor is present, i.e., if the parallel to the direction of motion through the centre of gravity of the particle does not intersect the obstacle.

Similar considerations apply to the case of bulk solid components of different specific weights.

The segregation of a bulk solid thus occurs as a result of various processes during the filling and emptying of a bunker.

Segregation phenomena impede the further processing or deposition of heaps of different granule sizes and changing specific weights. Segregation occurs during the temporary storage of such materials in many branches of industry.

1.2 Lines of research on bunkering

The reason why the Mining Institute Clausthal has specially concerned itself for some time with the problem of the segregation of heaps is that in its view only limited sucess has so far been achieved in the dimensioning of bunkers and silos in order to avoid segregation phenomena, with only a few exceptions, such as Jenike (Ref. 3), Kvapil (Ref. 4), Valentin (Ref. 5), the last mentioned using the theory of Jenike. Even the intensified efforts in the last ten years to obtain mathematical data have not yielded any major improvement. This is due mainly to the methods used.

In this connection a rough distinction can be made between three lines of research:

The first concerns itself exclusively with loose bulk solids. It endeavours - on the basis of the laws of soil mechanics - to determine all the possible characteristics before considering conditions in the bunker.

The second - to which most of the published works belong - is based on operating experience and results. It makes comparison and infers relationships from which recommendations can be drawn up for the reconstruction of units or the building of new and larger units. This method cannot be fundamentally rejected, but at the best it can only be approximately valid in an actual case, is fairly subject to error and does not yield any optimum dimensions for the bunker as long as the dimensionless factors which determine the movement process are not known.

The third line of research considers the bunker practically as a container from which stored material cannot flow under gravity alone. These may be either faulty constructions or bunkers with particularly large dimensions. This line develops bunker internals and bunker emptying systems to ensure that the stored material can flow out.

Since therefore the present state of knowledge is still unsatisfactory and the trend towards larger dimensions results in ever greater uncertainties, while at the same time a very careful and thorough experimental study of industrial plants of various sizes is impossible because of the time and expense entailed, an attempt will be made to show how the beginnings of a solution to the problem may be sought by a different route.

2. The essential characteristics in the bunker/bulk-solid complex

What are in fact the characteristics which determine the bunker/bulk-solid complex? Experience has shown that the type of bulk solid movement in a bunker depends upon the bunker geometry, the composition of the solids and the operative forces.

The factors listed in Table 1 influence the flow behaviour of bulk solids - which need not concern us at this point - and thus affect the segregation. Other essential factors are the type of charging, i.e., the manner in which the material to be stored in the bunker is put into the bunker, which is here denoted with A, and the discharge velocity w.

The segregation is a function of all these magnitudes. It can be described as follows:

$$E_f = f(H, D, d, \Theta, k, f_B, f_A, d^*, \bar{n}, \bar{\sigma}, \mu, \mu^*, P_R, P_S, P_T, \acute{A}, w, \ldots) \qquad (1)$$

Whether expressed in this way or as a power function, the complexity of the solution is obvious. It must be added that it is not certain whether all the factors influencing the segregation have been included, and that assumptions have been made concerning these factors whose validity or insufficiency can only be determined experimentally.

It would now be necessary to follow up with a mathematical formulation of the movement of the bulk solids, in order to reach a reliable result quickly. This operation is similar in importance to the determination of dimensionless characteristics, which of course are arguments in the differential quotients of the movement equation. If these arguments are known, then model tests can be performed to study the various movement phases of the bulk solids when segregation occurs (Ref. 6).

In the present state of knowledge considerable difficulty is encountered in any attempt to set up an equation of motion for the movement of bulk solids in a bunker, since the movement is yet insufficiently known. For this reason numerous and very careful experiments are necessary. In order to evaluate these and to reduce the number of tests and the size of the experimental facilities to a minimum, it is best to introduce dimensionless characteristics.

Now since no mathematical formulae are known for the movement equation for segregating bulk solids, in the case under consideration there is a special difficulty in finding the actual dimensionless arguments. Only one technique promises success, namely, dimensional analysis, the task of which must be to supply dimensionless arguments consisting of power products of the magnitudes describing the movement in the bunker.

The starting point is that the movement of the stored particles in the bunker is a dynamic process. This means that for a model bunker to function similarly to the full-size version, geometrical similarity alone is not sufficient, since together with the magnification ratio λ for the unit of length there are also the magnification ratios τ and κ for the times and forces respectively.

The initial equation of this dynamic phenomenon is represented in Table 1 as a functional characteristic. The function contains factors which are dimensional and also which are already dimensionless. If only the dimensional magnitudes are included, we have the following equation:

$$E_f = f(H, D, d, d^*, \bar{\rho}, P_R, P_S, P_T, w) \qquad (2)$$

This parameter equation partly characterizes the bunker problem. Since the process is dynamic, all the dimensionless characteristics contain three "determination magnitudes" for length, time and force, so that (n - 3) "characteristics" remain over. These in turn possess (n-3) "guide parameters" which determine the nature of each characteristic (Ref. 8).

The following are chosen as determination magnitudes:

for the force in /kp/	-	the force of gravity P_S /kp/
for the length in /m/	-	the diameter of the bunker D /m/
for the time in /s/	-	the discharge velocity of the bulk solids w /m s^{-1}/

3 determination magnitudes	-	P_S, D, w
6 guide parameters	-	H, d, d^*, $\bar{\rho}$, P_R, P_T

6 characteristics

$$K_1 = H \cdot P_S^{\alpha_1} \cdot D^{\beta_1} \cdot w^{\gamma_1}$$
$$K_2 = d \cdot P_S^{\alpha_2} \cdot D^{\beta_2} \cdot w^{\gamma_2}$$
$$K_3 = d^* \cdot P_S^{\alpha_3} \cdot D^{\beta_3} \cdot w^{\gamma_3}$$
$$K_4 = \bar{o} \cdot P_S^{\alpha_4} \cdot D^{\beta_4} \cdot w^{\gamma_4} \qquad (3.1 - 3.6)$$
$$K_5 = P_R \cdot P_S^{\alpha_5} \cdot D^{\beta_5} \cdot w^{\gamma_5}$$
$$K_6 = P_T \cdot P_S^{\alpha_6} \cdot D^{\beta_6} \cdot w^{\gamma_6}$$

The method of calculating the characteristics will be illustrated by the example of K_1.

Calculation of K_1

H in m^{+1}

P_S in kp

D in m^{+1}

w in $m\ s^{-1}$

If these units are inserted in equation (3.1) we obtain the following:

$$K_1 = m^{+1} \cdot kp^{\alpha_1} \cdot m^{\beta_1} \cdot m^{\gamma_1} \cdot s^{-\gamma_1} \tag{4}$$

In order for K_1 to become dimensionless, the exponents for force, length and time must beeome zero, i.e.,

$$\begin{aligned} &\text{for the force:} && 0 = \alpha_1 \\ &\text{for the length:} && 0 = +1+\beta_1+\gamma_1 \\ &\text{for the time:} && 0 = -\gamma_1 \end{aligned} \tag{5}$$

From these three equations with three unknowns it follows that:

$$\begin{aligned} \alpha_1 &= 0 \\ \gamma_1 &= 0 \\ \beta_1 &= -1 \end{aligned} \tag{6}$$

and we obtain the equation:

$$K_1 = H \cdot D^{-1} = \frac{H}{D} \tag{7}$$

The other magnitudes can be similarly calculated as follows:

$$K_2 = \frac{d}{D} \tag{8}$$

$$K_3 = \frac{d^*}{D} \tag{9}$$

$$K_4 = \frac{\bar{\rho} \cdot D^2 \cdot w^2}{P_S} \tag{10}$$

$$K_5 = \frac{P_R}{P_S} \qquad (11)$$

$$K_6 = \frac{P_T}{P_S} = Fr \qquad (12)$$

The symbolic equation with the six dimensionless characteristics now reads as follows:

$$f\left(\frac{H}{D}, \frac{d}{D}, \frac{d^*}{D}, \frac{\bar{\rho}\, D^2\, w^2}{P_S}, \frac{P_R}{P_S}, Fr\right) = 0 \qquad (13)$$

In order to refer the quotient of the forces of friction and gravity to a known expression it is advisable to expand $\frac{P_R}{P_S}$ with $\frac{P_T}{P_T} \cdot \frac{P_T}{P_R} = Re$ and $\frac{P_T}{P_S} = Fr$.

In place of $\frac{P_R}{P_S}$ and Fr in equation (13) we can now write the following:

$$\frac{Fr}{Re} = \frac{P_R}{P_S} = \frac{w \cdot \nu}{g \cdot 1^2} = \frac{w \cdot u\ \eta}{1^2 \cdot o \cdot g} \qquad (14)$$

$$E_f = f\left(\frac{H}{D}, \frac{d}{D}, \frac{d^*}{D}, \frac{\bar{o}\ D^2\ w^2}{P_S}, \frac{Fr}{Re}, \Theta, \frac{f_B}{f_A}, \bar{n}, A\right) \qquad (15)$$

Where it should be noted that the wall roughness k is included in the coefficient of friction μ^* for the bunker walling, and the granule shape f_K in the coefficient of friction μ for the bulk solids. The shape of the bunker and of the discharge aperture can be combined. It is then necessary to find what relationship exists between μ, μ^* and P_R. In equation (15) it was assumed that the coefficients of friction are contained in the frictional force. It must further be clarified whether the valid model law is to be found by way of Fr or Re or both characteristics, a question which, however, cannot be considered here.

If all the important parameters have been included in the initial equation (1), then equation (15) supplies all possible characteristics of the problem.

Of course, this statement requires experimental confirmation, especially since dimensional analysis together with the subjective choice of the determination magnitudes can provide a satisfactory solution only if all influential parameters have been included. A strict mathematical proof of this can scarcely be provided, but the proof that the factors have a physical meaning can already be evaluated as such (Ref. 6).

3. The preparation performance and evaluation of the experiment

The experimental set-up is determined by the mixture zones of the fractions of the bulk solids and the charging of this mixture into the model bunker. Fig. 1 shows the experimental installation in a somewhat simplified form. The two bulk fractions are mixed on their way from the hopper into the charging chute by collision with the bunker cover plate and the funnel walls and also, after a certain distance of free fall, by impact on the column of bulk solids in the charging chute, after which they are fed into the bunker by lifting up the whole charging facility. If it can be assumed that there are certain laws governing segregation on a cone, then it must be easy to check the quality of the mixture. An equal number of large and possibly small particles also must have become deposited on equally large areas of the bunker walling. By photographing and counting the granules it was proved that the input material was very well mixed. Depending upon the type of stored material and the diameter of the model bunker, a cone will be formed after a certain time which will rise up to the desired level in the bunker. The task now is to determine the granule distribution on individual samples or in the whole charge. For this purpose samples are taken at given intervals, for the parameter of prior interest is the composition of the product after it has been withdrawn from the bunker. Every sample is classified, i.e., divided into the original components, weighed and entered on a form.

In order to obtain information as to the "quality" of the mixture represented by each sample, the methods of statistical mathematics are generally used (Ref. 7). The standard deviation σ is determined. For a series of N/random samples we have:

$$\sigma = \frac{\sum_{i=1}^{N} \delta_i^2}{N-1} \qquad (16)$$

if it refers to the total quantity of the material studied. The δ_i values are the differences between the two components of each sample.

From the design of the above experiments and their evaluation a somewhat different definition was obtained of the distribution of the bulk solid components in the samples (Fig. 2). The segregation factor is defined as a ratio in the form:

$$E_f = \frac{\sum_{i=1}^{N} \delta_i}{G} \cdot 100 \quad /\%/ \qquad (17)$$

Why is it written in this way? This is determined by the type of evaluation, i.e., whether a differential or an integral result is sought (Fig. 2).

If no segregation were to occur during the whole bunkering operations then a fraction of 50% of each of the two components would show a vertical straight line at the 50% abscissa point.

If the origin is displaced parallel to the ordinate by 50%, the curve then shows the deficient granule quantity as a percentage of the sample quantity. If the deficient quantity is then plotted as a percentage of the total number of granules versus the bunker state, a curve is obtained whose area integral presents the total deficient granule quantity during discharge. Through the summation of all deficient granule quantities of both granule fractions a factor is obtained which describes the extent of the segregation during the bunkering process.

What difference now exists between the known standard deviation σ and the segregation factor E_f defined here? It is true that the former refers to the total quantity of the material studied, but it is only in the segregation factor that the weight of the total charge is introduced directly in equation. That is essential, since in the bunkering process the number of the sub-fraction withdrawn (= number of samplings for the σ deviation) is less decisive than their composition in relation to the total charge. The segregation factor thus contains the essential magnitude, and in addition is simpler to calculate than the σ deviation.

In order to facilitate a comparison in the evaluation procedure, Fig. 3 gives the values for E_f and σ for the same experiment. No notable differences appear.

4. Discussion of the first experimental results

As has been shown, the segregation factor is a function of all the characteristic magnitudes or a function of the dimensionless arguments set up.

We will give two examples to illustrate briefly the results obtained from dimensional analysis and the laboratory experiments.

Fig. 4 shows the influence of the ratio bunker level to bunker diameter on the segregation for various bunker sizes and shapes. Segregation increases with increasing bunker diameter and hence with decreasing d/D ratio. This result was foresseable because as D increases the particles acquire a greater freedom of movement, which is necessary for segregation to occur. If the results from bunkers of equal surface area but different shapes are compared with one another, the bunker with a rectangular cross-section shows the most marked segregation. Next come - as far as the maximum segregation factor is concerned - the bunkers of round and square shape.

If the segregation factor with changing H/D ratios is plotted versus the quotient of the diameter of the discharge aperture d and the bunker diameter D, then Fig. 5 shows clearly that E_f decreases both with increasing H/D and with increasing d/D. The result can be explained firstly from the fact that for constant bunker level and reduced bunker diameter the freedom of movement of the particles is reduced. In addition, as d increases with respect to D, a uniform drop in the column of material occurs which for d/d = 1 is only influenced by the wall friction.

The discussion of further results must await a further publication by the author.

After a description of the special conditions for a segregating heap in a bunker, an attempt was made by means of dimensional analysis to show how the experimental results to be found on the model could be transferred to the full-size version.

The results described consitute only a part of the complex of bulk solid movement. Other influences have already been analysed in the experiment but still have to be evaluated, while others again still require careful study.

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Notation	Legend	Unit	Dimension
	BUNKER GEOMETRY		
H	Height of bunker, maximum height of bulk solid	mm, m	1
D	Diameter of bunker	mm, m	1
d	Diameter of discharge aperture	mm, m	1
Θ	Angle of bunker floor to vertical	(-) arc.m.	(-)
k	Wall roughness	(-)	(-)
f_B	Shape of bunker	(-)	(-)
f_A	Shape of discharge aperture	(-)	(-)
	BULK SOLID DATA		
d^*	Granule size	mm, m	1
f_K	Granule shape	(-)	(-)
$\bar{n}$	Granule distribution (inclination d granule line in RRS diagram)	(-)	(-)
$\bar{\rho}$	Density, density differential	kp $s^2 m^{-4}$	$\rho\ t^2\ l^{-4}$
μ	Coefficient of friction for bulk solid	(-)	(-)
μ^*	Coefficient of friction for bunker walling	(-)	(-)
	FORCES		
P_R	Friction	kp	l w u
P_S	Gravity	kp	$l^3\ \rho\ g$
P_T	Inertia	kp	$l^2\ \rho\ w^2$
A	TYPE OF CHARGING		
w	DISCHARGE VELOCITY	$mm.s^{-1}$ or $m.s^{-1}$	ll^{-1}

$$E_f = f(H,D,d,\Theta,k,f_B,f_A,d^*,f_K,\bar{n},\ \bar{\rho},\ \mu,\ \mu^*,\ P_R,P_S,P_T,A,w,\ldots)$$

DETERMINATION OF LOADS ON SILOS BY MEANS OF AN INSTRUMENTED MODEL

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Abstract

Instrumented models were developed (base 70 x 70 cm, height 5 m) to enable all silo loads to be measured simultaneously, i.e., horizontal, wall friction and vertical loads. So far the measurements carried out on the models show that the exponential function on which the new German Standard DIN 1055 Bl. 6 is based is correct. It was proved, furthermore, that the discharge pressures are considerably higher than the filling pressures but that the maximum pressure curve can also be represented by an exponential function. Silo loads were found to be independent of the rate at which the silo is filled and emptied. On the other hand, the silo loads proved to be heavily dependent on the location of the discharge orifice within the cross-section. Wall roughness is an important factor only if it becomes high in relation to grain size. It was demonstrated that the roughness of corrugated sheet walls is so great that wall friction corresponds approximately to the angle of internal friction.

The behaviour of powdered solids, which produce very high pressures on quick filling, was investigated. As the material settles, however, the silo pressures fall very sharply so that for this condition the specifications of the German Standard are somewhat too high.

The pressures occurring when powdered solids are homogenized with air were also measured. The homogenization pressure corresponds to the fluid pressure of the powder/air mixture.

Measurements were also performed on the large pressure stresses arising on the collapse of internal arches such as form in cement clinkers when stored in silos.

1. Introduction

The silo research station in Braunschweig is responsible for checking the new German Standard DIN 1055 Bl. 6 for silo loads by amassing the most extensive experience possible of forces produced by bulk material and acting on the silo.

2. Test models

So far two instrumented model silos have been developed for this purpose. One has a square cross-section of 70 x 70 cm and is 500 cm high. Its side walls consist of a total of 40 independent, statically supported panels (Fig. 1). The bearing pressures from each panel can be measured with an accuracy of about 3% by means of calibrated gauges. Thus all the silo loads over the area of the panels can be measured simultaneously and integrated with good accuracy.

This silo is used to study the properties of granular bulk materials, the influence of wall roughness, the rate of filling and emptying and the influence of the position and shape of the discharge orifices. The influence of the cross-sectional shape can be studied by using inserts to vary that shape.

The second instrumented model silo is a thin-walled, suspended aluminium tube 80 cm in diameter and 600 cm high (Fig. 2). The measurements on this tube are performed by means of a large number of cemented-on strain gauges, and it is used to study powdered solids. Furthermore, it has extremely smooth walls and is equipped with a compressed air system which enables granular material to be aerated and powder to be discharged pneumatically and homogenized. The accuracy with this model is not quite so good, and errors of ±10% are to be expected.

3. Model scale

The most important question regarding such measurements is that of the ability of the test results to be transferred from the model to the dimensions of actual silos, which may be up to 40 times greater, in other words the question of model scale.

If we consider the formula for the horizontal load, which was developed by Janssen and is still regarded as valid today:

$$p_h = \frac{\gamma . F}{\mu . U} (1 - e^{-z . \mu . \lambda . U/F})$$

it is apparent that, on the one hand, the pressure is a function of the physical constants, namely the bulk density γ, the ratio of the horizontal pressure to the vertical pressure, i.e., $\lambda = \tan \rho$, if ρ denotes the angle of repose, and the wall friction factor $\mu = \tan \rho$, which is determined by the wall roughness and the internal friction conditions in the material. Both the material and the wall surface can correspond to reality in the relatively large test silo.

The second determining factor for silo loads is a shape factor, namely the ratio of silo cross-sectional area to circumference F/U. This value allows the formula to be adapted to any size and shape of silo provided that the ratio silo diameter/grain diameter does not become too small.

If the validity of Janssen's formula can be confirmed, therefore, there is no further model law when the same materials are involved, and the measured data can be transferred directly from the model to actual practice using Janssen's formula.

In order to study these questions experiments were carried out in various silos with the same material, namely, quartz sand with a grain diameter of 1-2 mm, and almost identical wall surfaces. If the above observations were correct, then all the measurements, when converted via F/U, would give the same result. So far, experiments have been performed with two cylindrical silos and one square one. Fig. 3 shows the result. The curves for the filling pressures agree very closely, whereas those for the discharge pressures display a slight scatter.

Complementary experiments are planned with rectangular silos and a square silo having a 100 x 100 cm cross-section. A comparison with published measurements made on silos in use has also confirmed the transferability of the measurements made on the models.

4. Influence of wall roughness

The wall friction angle δ has a very strong influence on the silo loads. It determines the horizontal load at infinite depth.

Its magnitude varies greatly, depending on whether the material is static or flowing. The friction angle during flow is considerably less than during a state of rest. The main difference between the old method of calculating a silo design and the new Standard lies in the recognition of this fact. This also explains the pressure-rise factors observed by Reimbert during emptying as compared with filling.

Compared with this influence that of wall roughness is relatively insignificant. As Fig. 4 shows, during the tests with brewer's barley the plywood wall of the test silo was made smoother from one experiment to the next, and the final filling pressure of $\delta = 0.75$ was only attained after about ten flow cycles.

No difference could then be observed between this smoothed wall and the very smooth wall of the aluminium tube.

Wall roughness is obviously not so much a matter of major unevennesses as of a surface that behaves as though lubricated, such as can be observed in any grain silo.

The observation that any wall becomes polished in service justifies the specifications in the Standard, in which the wall's angle of friction is made independent of the wall material and is solely a function of the angle of internal friction of the stored material. The tests carried out to date also confirm the figures specified in the Standard as being $\delta_f = 0.75\rho$ for the wall's angle of friction at rest during filling and $\delta_e = 0.60\rho$ for the wall's angle of friction in motion during discharge.

The values deviate if the ratio of the size of the asperities to the material grain size is very high. This arises when, for example, the grain size is very small, as in the case of powder, or when there are very coarse, regular "asperities" such as are produced by corrugated sheet mounted transversely (Fig. 5).

In both cases the irregularities can be thought of as being filled by the bulk material, so that the plane of motion runs through the material. The wall's angle of friction during filling is then about equal to the angle of internal friction, whereas that during emptying is 0.75ρ. Since the wall friction loads are larger, the horizontal loads are considerably reduced, so that it is possible subsequently to relieve the loads on silos that were built too weak.

5. Influence of the pressure coefficient

The ratio λ of horizontal to vertical pressure determines the "fullness" of the pressure curves, i.e., the extent of the rise in the horizontal pressure from zero to the maximum pressure with the filling height. Formerly, the active earth pressure coefficient was used for this value, after Koenen. This value has proved to be wrong; indeed it could not have been right, since it is not the active earth pressure but the static earth pressure that must prevail during the filling of the silo. The static pressure coefficient is given in the Standard as being roughly $\lambda_f = 0.50$. From the shape of the filling-pressure curves it can be seen that this value is somewhat too high.

Unlike the filling curve, the discharge-pressure curve does not represent a state but is the envelope of a large number of state curves. For this reason the quoting of a coefficient has in this case only a descriptive and not a physical significance. According to the latest measurements the pressure coefficient for the discharge curve is about $\lambda = 0.50$ throughout. In the light of contemporary knowledge the Standard deliberately specified the high value of $\lambda = 1.0$ in order to cater for the supplementary stresses due to uneven filling which occur in the upper region and cannot be calculated (Fig. 6).

6. Ifluence of the filling and discharge rates

It was observed during the first experiments at Braunschweig that the rates of filling and discharge do not affect the silo stresses when the discharge orifice is centrally situated. The experiments were repeated with brewer's barley and again showed no representable dependence (Fig. 6). Since the rates in the experiments were in any case multiplied by a factor of ten, it can be assumed that this observation is also applicable to large silos.

Only in experiments with slot-shaped orifices at the edge (Fig. 10) was the rate of filling found, surprisingly, to exert any influence, not, indeed, on the magnitude of the silo pressure but to a very marked degree on the form of the pressure curve.

It should be mentioned here that all the pressure curves shown are the mean values from at least three experiments, so that random factors or errors in measurement are ruled out.
The influence described must be gone into more fully.

7. Influence of an eccentrically located discharge orifice

It has been known for some time that eccentrically located discharge orifices give rise to quite different emptying pressures from those that occur when the orifices are centrally located. It is also known that almost all seriously damaged silos had non-central orifices and that the pressures with eccentric orifices must therefore be greater. Reliable measurements relating to this phenomenon have now been made for the first time.

Fig. 7 shows the first result of the experiments with quartz sand, i.e., the fact that the greatest pressure is not created by the extremely eccentric discharge orifice but that which occupies a semi-eccentric position.

Fig. 8 shows the differences in the pressure evolution in the silo walls which are close to and those which are distant from the discharge orifices. The highest pressure occurs on the discharge side; on the opposite side there is hardly any change in the pressure as compared with a central orifice. Fig. 9 which is based on the experiments with durum wheat, shows, however, that this observation does not hold good for all materials. With the wheat the pressure on the discharge side and that on the opposite side vary in almost the same way, although not to an equal extent.

The data in the Standard to the effect that the pressure decreases on the discharge side and increases on the opposite side are therefore in need of revision. It would be advisable to assume initially, that the pressure increase is equally large on all sides. The amount of the pressure increase is fairly accurately estimated in the Standard; it is in the region of 30%.

Fig. 10 shows the pressure curve during discharge through a slot running along the wall (five separate orifices). The pressure increase as compared with the case of central discharge is 40%, i.e., much greater than that specified in the Standard. The results lead to the conclusions that during discharge through a peripheral slot the horizontal stress on the silo walls will be even higher. Caution is therefore necessary with such designs.

8. Behaviour of powdered solids

When powder is being poured in it entrains at first a quantity of air and hence flows with hardly any internal friction, like water. Almost flat surfaces are created and the pressure rise corresponds to that of a liquid with the same density as the loose powder.

If this bed is left static it gradually settles, with a consequent rise in the internal friction and, probably, in the cohesion too. In a powder these two effects cannot be distinguished. The result is a rapid drop in the horizontal pressure. Fig. 11 shows the pressure falls with time in the case of cement.

Fig. 11 shows how the pressure falls with time in the case of cement. Fig. 12 shows the same process in powdered limestone and Fig. 13 in wheat flour. The pressure curves as plotted for comparison in the Standard are therefore too high if the level of the powder in the silo rises only slowly, but yield values which are too low if the silo is filled very quickly, as can occur, for example, when a homogenizing silo is emptied into a storage silo beneath. Fig. 14 gives the curve for the pressure occurring during homogenization, i.e., when enough air is injected for all the powder to become turbulent and thus mixed. The filling-pressure curves shortly before air injection are shown at the bottom of the figure. Injection was started at various times after charging with cement, which accounts for the great variation in the curves. This has no influence on the homogenizing pressure, which is somewhat lower than that laid down in the Standard.

9. Properties of various bulk materials

Together with general findings, the foregoing test data provide information on the behaviour of various bulk materials. Cleaned barley seems to produce the greatest pressure of all cereals. The measured values are some 17% higher than those specified in the Standard for cereals generally, and this should be taken into account in future. Quartz gravel behaves considerably more favourably than is specified in the standard for gravel in general. This type of gravel was selected for all the experiments in which the material properties had to be the same because it undergoes no considerable change even after many passages through the silo and because further supplies of the same quality can be obtained cheaply at any time.

Experiments with cement clinker have shown that the properties of this material are appreciably more favourable than is stated in the Standard. They may be taken as $\gamma = 1.5\ Mp/m^3$ and $\rho = 35°$

10. Effects of dynamic arches

A rhythmic pulsation of the flow, known as "pumping", occurs in many silos. Other authors have several times measured its effect on the horizontal pressure as being not more than 10%. It is probably due to aeration phenomena in the flowing material and not to the collapse of genuine arches.

During experiments with a particular type of cement clinker, however, pressure variations were measured which can only be explained by such collapses (Fig. 15). The three pressure diagrams reproduced are typical extracts but do not relate to the same period of time.

At level 3 there were only pressure rises in all the experiments, whereas at the higher levels the pressure rise was preceded on each occasion by a very sharp but very brief pressure drop.

If it is assumed that this pressure drop occurs when the material falls into the space beneath the collapsing arch and therefore loses contact with the walls, while the pressure rise results from the deceleration of the falling masses, then the obvious conclusion is that the arch must have formed in the region of panel 4. It is not yet known why the arches always formed in this particular zone, and why they occurred only when there was at least 250 cm of material above this level. The shape and size of the arches and their dependence on the silo cross-section and the depth of overlying material are likewise still unknown.

However, the sudden pressure rise from about 0.5 p_f to 2.0 p_f warns of the need for caution in calculating the stresses for silos for such materials. In addition to the absolute pressure level, account should be taken of the danger of fatigue failure due to the very frequent cyclic loads.

11. Summary

The results presented here only partly reflect a very comprehensive series of tests which are still in progress. In the diagrams we have dispensed with showing the values for the wall friction load and floor load which were measured simultaneously with the horizontal pressure. These correlated series of measurements, together with the scope of the results, are particularly valuable for the recognition of the physical interrelationships.

The results of measurements made so far show the practising engineer that the data furnished in the Standard for the calculation of the loads on silos are not altogether correct in certain details. They also show, however, that the principal characteristics can be calculated entirely in accordance with the Standard. With these assumptions regarding loads, silo failures will not occur.

To some extent we have made use here of results which cannot be published in theses for some time yet. This "pre-publication use" is necessary in the interests of practical application; it should not detract from the originality of the thesis.

Fig. 1 Design of the prismatic model silo

Fig. 2 Bottom of the cylindrical model silo

Fig. 3 Filling and central discharging; 1-3 mm quartz sand; horizontal load p_h on the cylindrical model converted for the prismatic model with the aid of the ratio F/U

Fig. 4 Filling: brewer's barley; wall friction load p_w and horizontal load with smoothing of the wall and with moisture content

Fig. 5 Filling and central discharging; brewer's barley; corrugated sheet and plywood walls

Fig. 6 Filling and central discharging; brewer's barley; increasing the rates of filling and discharging by a factor of ten had no discernible effect

Fig. 7 Discharging via an eccentric point-orifice; 1-2 mm quartz sand

Fig. 8 Discharging via an eccentric point-orifice; 1-2 mm quartz sand

Fig. 9 Discharging via an eccentric point-orifice; durum wheat

Fig. 10 Discharging via an eccentric slot-shaped orifice; 1-2 mm quartz sand

Fig. 11 Cement; variation of p_h with f(t)

Fig. 12 Powdered rock; variation of p_h with f(t)

Fig. 13 Wheat flour; variation of p_h with f(t)

Fig. 14 Cement; level: 3.0 m, followed by homogenization

Fig. 15 Horizontal silo load with arching; bulk material: cement clinker $\gamma = 1.5$ Mp/m^3, $\rho = 32°$, $\rho_e = 19°$

SHEAR EXPERIMENTS ON PEBBLE BEDS

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Abstract

This contribution briefly reviews the theoretical and experimental studies on the mechanical behaviour of regular spherical packings carried out at the Institute for Soil and Rock Mechanics of the University of Karlsruhe. A few selected examples illustrate the mechanical properties of these packings under strain deformation as revealed in the laboratory by soil testing methods such as three-dimensional or plane shear and static pressure. Comparisons between experimental results and theoretical characteristic curves confirm the validity of the analysis findings.

1. Introduction

The work done at the Institute for Soil and Rock Mechanics of the University of Karlsruhe on the mechanical behaviour of pebble beds is no doubt only indirectly related to the problems of the THTR project, since our studies refer mainly to a pebble bed model with a radically simplified geometry (regular single-sphere packings, shown in Fig. 1), which furthermore are subjected to a very simple stress condition, the homogenous main stress condition.

The final aim of our work is to provide a more accurate description of the strain-deformation and rupture behaviour of non-cohesive soils such as gravels or sands, and to determine the relevant physical parameters qualitatively and quantitatively. The choice of regular spherical models enables such studies to be conducted with the required physical and mathematical strictness. It must, however, be recognized that it also makes it impossible to take all parameters into consideration from the outset.

These studies cannot be expected to have any direct application to practical soil mechanics, but they do make it possible to study with considerable precision the influence of the individual particles and their arrangement on the mechanical behaviour of a total structure built up of individual particles, using only the basic physical properties of elasticity and friction and avoiding assumptions as to essential mechanical properties of the bed as a whole.

What follows is a summary of some results obtained in our Institute on the mechanical behaviour of pebble beds. All the relationships shown are based on analytical proofs to be found in the appropriate numbers of the Institute's publications. Since only a short review can be given here, the formulae and proofs are omitted.

2. The geometry of pebble beds

First some comments on the geometry of the pebble beds shown in Fig. 1. These consist of layers with either a square or triangular mesh of sphere centres. The mesh pitch is arbitrary only within certain limits. The finest mesh occurs when the spheres of a single layer touch one another and the widest when the intervals between the spheres of a single layer are so large that the spheres of the next layers above and below touch at their top and bottom points.

The characteristic taken for the packing geometry within the above-mentioned limits is the angle of contact j, which in the regular packing is the same at every point of contact. This is the angle between the perpendicular to the sphere surface at the point of contact with a sphere of the next higher layer and the horizontal.

The bottom diagram in Fig. 2 shows the correlation between the pore volume n of the spherical packings and the angle of contact j. The pore volume is a maximum for a certain angle of contact which is not equal to the respective limit angles; with both types of array this is the angle j = 35.3°. In contrast to the angle of contact, the pore volume is thus not a clear indication of the geometry of a regular packing.

3. Strain-deformation relationships of pebble beds

One example of the mechanical properties of a spherical lattice of this kind which interest us is the main strain ratio $\sigma_1/\sigma_3 > 1$ up to which a regular spherical packing can be loaded if the coefficient of friction of the spheres has a given value f. It is first necessary to decide what strain or deformation conditions are to be considered. In a standard experiment in which a cylindrical sample is first subjected to uniform all-round pressure and then stressed axially to rupture, Wittke (Ref. 1) has developed the relationships shown in Fig. 2 on the basis of an initial concept by Idel (Ref. 5).

The relationship of the critical main strain ratio to the angle j is thus the same for both types of array - square and hexagonal, but the pore volumes may differ greatly. The critical main strain ratio is greatest for the greatest possible angle of contact, i.e., for the packing with the smallest pore volume which is the densest packing. As the angle of contact diminishes, the ratio $(\sigma_1/\sigma_3)_{crit}$ decreases monotonically. If the curves for various values of the coefficient of friction f are compared, it is seen that the main strain ratios which can be supported by the packing increase with increasing inter-sphere friction, as is to be expected. From the curve for f = 0 it is clear that the shear strength of such a packing does not depend solely upon this inter-sphere friction. However, as well as the type of array, the material friction coefficient is also an important factor influencing the supporting capacity of spherical packings and also other properties mentioned later. In all questions on the mechanical behaviour of pebble beds it is therefore especially necessary to ascertain clearly the size of the coefficient of friction f. But in our experience this is extremely difficult, even if one is aiming only to be accurate to within a few tens of per cent.

It can also be deduced from the basic relationships of Fig. 2 that the critical main stress ratios assume an infinite value for sufficiently large coefficients of friction between the spheres. With the densest hexagonal array this occurs with f values of 0.707 and above. This means that the vertical load σ_1 on a packing - independently of the lateral pressure and even in its absence - can be increased indefinitely with sufficiently large material coefficients of friction without causing rupture of the packing; the possible crushing of an individual granule can thus be disregarded.

This can be seen with the aid of the plane four-sphere model shown in Fig. 3. If the material coefficient of friction f is larger than or equal to the tangent of the angle (90° - j), the load P without supporting force P' can be increased indefinitely, without the two central spheres being forced apart.

The freestanding wall of sanded glass spheres is shown on the right of Fig. 3. The spheres are not bedded together in layers as in Fig. 1, for example. The model is admittedly subjected to a plane strain condition (see Fig. 1), in which smaller material coefficients of friction are required (see Fig. 9) for $(\sigma_1/\sigma_3)_{crit} = \infty$ than in the three-dimensional strain condition, but the effect shown on the left of Fig. 3, which forms the basis of the free supporting capacity of the packing is the same in both cases.

If it is assumed that the spheres of the packings considered are non-deformable, the relationship shown in the upper diagram of Fig. 2 can be interpreted as a strain deformation law. This may be illustrated by the example of the coefficient of friction f = 0.2. Let an extremely dense hexagonal packing with f = 0.2 be initially exposed to a uniform all-round pressure. In the diagram point 1 corresponds to this. The packing can then be loaded up to the critical main stress ratio - here 6.37 (point 2) - without the angle j altering; in this state the friction possible at the points of contact is overcome, the layers of the packing begin to push in upon one another, and the angle of contact diminishes. The resistance offered by the packing to the axial compression diminishes as shown by the thick solid line for f = 0.2. It is still necessary to determine the percentage compression of the spherical packing corresponding to the alteration of the angle of contact; this can easily be done by purely geometrical methods and is indicated by the upper abscissa scales of the diagram for the two densest packings.

The question now is whether this analytical result does in fact describe the actual experimental behaviour of such a packing. In reply it can be said that the experiments conducted by Wittke have proved the influence of the type of array, the angle of contact and the coefficient of friction.

The main result of these earlier experiments was the abandonment of the assumption of non-deformable spheres, which results in the vertical pattern of the first section 1 2 of the curve in Fig. 2. Soils such as spherical packings are known to behave in such a way that the largest permissible load is attained only after certain axial compressions.

In other studies therefore (Ref. 2), allowance was made for the elastic behaviour of the spherical material in setting up the stress deformation equations of spherical packings. The result may be illustrated by the densest square packing. In Fig. 4 the packing deformations which occur with elastic spheres are plotted up to the maximum possible value of the main strain ratio. The curves shown here are thus those which appear as vertical lines in Fig. 2 (e.g. 1' 2') when the spheres are considered as rigid bodies.

The abscissa of the diagram in Fig. 4 is so chosen because the deformations of the packing are proportional to the 2/3 power of the lateral pressure σ_3.

The curves will not be discussed in detail here. Mention should merely be made of the fact that the spherical packing behaves the more rigidly the greater the coefficient of friction f. It may also be noted that the deformations as a whole are very small until the supporting capacity of the packing is reached. Even for $\sigma = 30\ kg/cm^2$, ε_1 is only in the order of magnitude of 4×10^{-4} for the elasticity properties of glass or quartz.

If the strain deformation relationships from this diagram and the diagram in Fig. 2 are put together, a curve is obtained which should represent the behaviour of such a spherical packing in the triaxial experiment. It is permissible simply to couple the results for elastic and rigid pebbles if one is considering only a small zone of the strain change after attainment of the maximum bearing force, a zone within which the deformation components are only negligibly altered by the deformability of the spheres.

Fig. 5 shows a comparison between experiments and the theoretical characteristic curve obtained in this way. It can be seen that the result of the analysis almost exactly reproduces the experimental behaviour. This illustrates the fact that a satisfactory description of the strain deformation behaviour of a regular packing can be achieved by analytic methods. We would also mention that the curve of the dependence shown here corresponds qualitatively with the strain deformation behaviour of dense unfixed materials in the triaxial experiment.

Another question which has become relevant recently in connection with the shear strength of unfixed earth is that of the influence of the strength of the individual granule on the supporting capacity of a packing.

The parameters concerned in this problem can be studied analytically on spherical packings (Ref. 2). By making certain assumptions as to the rupture danger of two spheres in contact with one another, it is possible to determine in what way the critical main stress ratio depends upon the magnitude of the lateral pressure. While the condition that the spheres slide over one another leads to a maximum possible main stress ratio independent of the lateral pressure, only minor loads can be borne by the spherical packing above a certain limit if the strength of the spherical material is limited. The theoretical curve plotted in Fig. 6 is compared with the results of a series of experiments on glass spheres in the lateral pressure range of $\sigma_3 = 6 - 100$ kg/cm^2. The analytic result obtained under simplified assumptions reproduces the experimental behaviour of the spherical packings very satisfactorily, as we know.

By tests on irregular beds of small glass beads it was shown that the relationships studied on the regular structure play a decisive role in such packings also.

As can be seen from Fig. 7, where the results of this series of experiments are plotted, the experimentally derived relationship shows the kink typical of the theoretical characteristic curve, this kink being linked with the occurrence of rupture of individual granules. This is illustrated by the rupture which occurred during the experiments (line top right in Fig. 7). Owing to the irregular arrangement of the glass beads it was possible to make here only a qualitative comparison by confronting the experimental results and the theoretical findings obtained for the various types of array from the loosest to the densest.

These results account for the fact that the important shear strength parameter of the so-called angle of internal friction of an unfixed earth is not a constant independent of the load.

4. Static pressure in pebble beds

A second important problem in connection with the mechanical properties of packings is that of the static pressure. By this we understand the pressure which a homogeneously loaded packing exerts laterally on the frictionless walling of a rigid container, i.e., in the state of impeded lateral expansion. A work by Raju (Ref. 3) on this subject showed that the static pressure coefficient λ_o, which is defined as the ratio of the static pressure to the applied load, is equal to the coefficient which obtains in conditions where there is no lateral impediment to expansions. It is assumed that the packing is not pre-loaded. This means that the main stress ratio in the thus defined static pressure state is the same as that in the triaxial experiment for the same spherical packing in the rupture state.

This result can be illustrated physically on a simple plane model of three spheres, as shown in Fig. 8. The condition of impeded lateral expansion of the packing corresponds here to the rigid embedding of the two lower spheres in a block. On being loaded, the upper sphere (C) transmits the force P via normal and tangential forces to the two lower spheres (A, B). The path of the sphere centre C, which for reasons of symmetry must be vertical, is then composed of a normal and a tangential displacement component with respect to the centres A and B. The elasticity of a sphere with respect to tangential displacements is so small by comparison with its elasticity with respect to normal displacements and with common materials that the compatibility of the displacements can be attained only through sliding at the contact points. But this means simply that at the contact points of a spherical packing subjected to a load for the first time, the full possible friction is activated even when lateral expansion is prevented. Raju was able to demonstrate the validity of his results by experiments on packings of steel spheres in the densest type of both forms of array.

The above- mentioned studies are concerned with the behaviour of spherical packings in the special case of the triaxial stress state, in which the two smaller main stresses are equal to one another. Deformations are then either equally possible in both directions of the smaller main stress, or are impeded. In connection with the first systematic studies on the mechanical behaviour of spherical packings Wittke studied another deformation condition, known as the plane strain condition. A spherical model in the plane strain condition ahs already been shown in Fig. 1.

The analytical treatment of this case shows (see Fig. 9) that given two equal small main stresses the changed deformation alone enables a packing to support heavier loads in the plane strain condition than in the three-dimensional state, and that the deviation from the three-dimensional condition increases with the material coefficient of friction f. Only when $f = 0$ do both conditions result in the same supporting loads or critical main stress ratios. This result confirms and partly clarifies the fact that the angles of internal friction measured on earths in plane models are greater than those found in the three-dimensional model.

5. Influence of disturbances

All the above results of studies on the mechanical behaviour of packings assume an ideal regular array. Hence arrangement between theory and experiment could only be demonstrated for spherical packings which approximated to the ideal model. The above experiments were therefore conducted with packings of spheres of very slight irregularity and diametrical tolerance (a few μ).

Our early experimental studies with much less regular spheres, on the other hand, led to results which agreed broadly with theory but were quantitatively unsatisfactory. In contrast to the above works, in which fault-free spherical models and load conditions were assumed, Weseloh (Ref. 4) therefore recently studied a spherical packing in the plane strain condition with the assumption of disturbing influences. Using energy methods he shows by stability studies that the rupture of a spherical packing which contains packing faults (e.g., irregularities in the spherical shape) or faults in the load condition occurs not because of uniform lattice deformation but on distinct shear surfaces or zones; individual granules in the shear zone undergo rotary movement as shown on the right of Fig. 10 [x)].

On the left in Fig. 10 the resulting critical main stress ratio is compared with that which occurs in the absence of faults and is plotted as a function of the material coefficient of friction.

The upper line, which applies to the absence of faults, corresponds to that shown in Fig. 9 for the densest square packing in the plane experiment.

In general, samples which tend to this sort of shear zone formation have critical loads appreciably lower than those which can be withstood by samples whose loads are as far as possible fault-free. With increasing material coefficient of friction the difference in the supporting capacity increases noticeably; for $f = 0$ the influence of the fault on the critical load disappears. As can be seen from the entries on the diagram in Fig. 10, the behaviour of moderate-precision spherical packings can be satisfactorily described by analytical result.

These are, in brief outline, some results of the studies carried out hitherto in the Institute for Soil and Rock Mechanics of the University of Karlsruhe on the mechanical behaviour of spherical packings. They may lead to a better understanding of the phenomena in simply loaded regular spherical packings. For the sake of completeness, some works are mentioned in the list of references which deal with the mechanical properties of peoole beds but have not been mentioned individually above.

[x)] These quite different considerations afford a parallel to the observations on shear zones in earth packings made by Professor Roscoe in his contribution.

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CHARACTERIZATION OF PACKINGS

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Abstract

Between the structural properties of a packing and the geometric properties of a section through it there exist functional relationships which can be deduced for the case of a uniform random arrangement of the granules.

1. Introduction

In many technical processes the solid material is in the form of a granular packing through which pass mass and energy flows whose quantities depend upon the structure of the packing. Knowledge of this structure is therefore necessary.

In order to characterize a packing of granules of various kinds it is necessary to have data on the packing density, the mixture state and the granule arrangement.

The packing density can be determined quantitatively with the solid volumetric component $(1 - \varepsilon) = V_s/V$ or the void fraction $\varepsilon = V_H/V$; V_s is the volume of the solid, V_H the void and V the volume of the packing.

Data on the mixture state are required with packings of granules of varying physical properties. Such packings always have a tendency to separate, and often actually do so.

The granule arrangement may be either regular or irregular. Among irregular arrangements the special case of the "uniform random arrangement" is distinguished from the others. It will be fully discussed below.

Figs. 1, 2 and 3 show sections through various packings for illustration purposes. Fig. 1 is a section through a packing of lead spheres 0.97 to 1.08 in diameter. There is an evident regularity, characterized by the fact that major part volumes can be represented by transposing a small unit volume. In general, regular arrangements occur only with packings of spheres of the same size. The build-up must also take place in a field of force and with sufficient energy to increase mobility. The spheres try to form the densest packing, which has the smallest potential energy compared with the others.

The regular arrangement can be avoided by a careful, low-energy structure. Fig. 2 shows a section through a packing of this kind, which also consists of lead spheres 0.97 to 1.08 mm in diameter. If spheres of various sizes, or non-spherical granules are used, the arrangement is predominantly irregular. Regularity does not occur because the forces of attraction and repulsion are not direction-dependent, and have ranges which are small by comparison with the granule dimensions. Magnetized granules or granules with electrostatic charges may possibly be exceptions to this rule. Fig. 3 is a section through a packing of limestone granules with diameters of 0.1 to 1.1 mm.

With packings of dimensions much greater than those of the granules, the three properties packing density, mixture state and arrangement will in general be place-dependent. Further data therefore include the magnitude of the homogeneity regions. Widely varying possibilities can be imagined. For example, a packing of granules of various sizes may be at once homogeneous and very loose, since given the same arrangement the packing density is independent of the granule size. A packing of spheres of the same size may have a regular arrangement in the lower part and near the wall, but an irregular arrangement in the upper part. In general it may be said that knowledge of one of the three properties named and of its homogeneity does not permit any inference as to the other two properties and their homogeneity.

The "section method" of studying the packing structure has proved valuable. The empty spaces are filled with fluid which sets, after which the packing is cut open (see Figs. 1-3). The sectional view can be measured according to the solid surface component $(1 - \varepsilon_F)$, the mean granule cross-sectional area $\bar{f}$, the mean diameter $\bar{\Theta}$ of the granular sections and the numerical distribution of the Θ. In order to draw conclusions from these data it is necessary to find out how they depend upon the packing properties and the granule magnitude distribution. For this purpose it is necessary to consider the two special cases of the uniform random arrangement and the regular arrangement separately.

2. Numerical distribution density of the cross-sections with uniform random arrangement

A uniform random packing can be defined as follows:

Let there be a method of making an irregular packing such that with any number of repetitions the probability of a granule centre of gravity occurring is equally great in every spatial element of the packing, and every orientation of the granules is equally probable.

Let the packing be built up of irregular granules. Let us agree to attribute to each granule a linear magnitude x; let n(x) be their numerical distribution density. A section results in surfaces of irregular shape. We call their magnitude f; $z^*(f)$ is the numerical distribution density of the f. Let it be further agreed to assign a nominal diameter Θ to the cross-section such that

$$\Theta = (4f/\pi)^{1/2} \tag{1}$$

the distribution density of the nominal diameter being $z(\Theta)$. $Z^*(f)$ and $z(\Theta)$ are to be calculated from n(x) on the assumption of a uniform random arrangement. For the solution it is necessary to determine the probability dW(f) of the occurrence of a cross-section of area f...f + df, since the equation

$$dW(f) = z^*(f)\ df \qquad (2)$$

applies.

Let dW_1 be the probability that the cross-section is due to a granule of magnitude x...x + dx, and

dW_2 the probability that the cutting of a granule of magnitude x results in a cross-section of size f...f + df.

Every granule has a maximum cross-section, so that for every given cross-sectional area f there exists a lower limit of grain size; this is called x_u. Let the maximum grain size of the distribution be called x_{max}. On the basis of these definitions the equation for determining dW(f) is:

$$dW(f) = \int_{x_u}^{x_{max}} dW_1 \cdot dW_2 \qquad (3)$$

For granules of equal size $dW_1 = 1$ and integration over x becomes unnecessary, so that we get the simple equation:

$$dW(f) = dW_2 \qquad (4)$$

Fig. 4 is a sketch of an irregularly shaped granule. In order to determine orientation, we agree on a certain diameter through the centre of gravity S. The position of the granule can then be described by means of the two angles ϕ and Θ, where Θ is the angle between the given diameter and the perpendicular of the cross-section plane. A granule is cut only if the cross-sectional plane lies between the tangent planes T_1 and T_2 of the granule which are parallel thereto. The distance t between T_1 and T_2 is dependent upon ϕ, Θ and the granule shape parameters ε_i as well as on x; these in turn are generally functions of the granule size: $t = t(x, \phi, \Theta, \varepsilon_i(x))$.

In order to determine dW_1 let us further assume that a mean granule form can be attributed to the granules of the fraction x...x + dx, thus giving us in principle the function $t(x, \phi, \Theta, \varepsilon_i(x))$. Without this assumption the general equation becomes even more awkward and unmanageable.

Let N^* be the mean number of granules per unit volume. The mean number of size x...x + dx with the orientation $\phi \ldots \phi + d\phi$ and $\Theta \ldots \Theta + d\Theta$ per unit volume is $(N^*/4\pi)\ n(x)\ \sin\Theta\, d\Theta\, d\phi\, dx$. The mean number per unit surface of sections of granules of this kind is $(N^*/4\pi)\ n(x)\ t(x, \phi, \Theta, \varepsilon_i(x))\ \sin\Theta\, d\Theta\, d\phi\, dx$. After integration over ϕ and Θ we obtain the mean number dN_s of cut granules of size x referred to the surface. From further integration with respect to x we obtain the mean number N_s of the cut grains with respect to the surface.

We then have:

$$dW_1 = dN_s^*/N_s^*$$

$$dW_1 = \frac{n(x)\ dx \int_{\phi=0}^{2\pi} \int_{\Theta=0}^{\pi} t(x, \phi, \Theta, \varepsilon_i(x))\ \sin\Theta\, d\Theta\, d\phi}{\int_0^{x_{max}} \int_{\phi=0}^{2\pi} \int_{\Theta=0}^{\pi} n(x)\ t(x, \phi, \Theta, \varepsilon_i(x))\ \sin\Theta\, d\Theta\, d\phi\, dx} \qquad (5)$$

Equation 5 is considerably simplified if the function $t(x, \phi, \Theta, \varepsilon_i(x))$ can be represented by the product $t_1(x)\ t_2(\phi, \Theta)$

$$dW_1 = \frac{t_1(x)\ n(x)\ dx}{\int_0^{x_{max}} t_1(x)\ n(x)\ dx} \qquad (6)$$

When there is geometrical similarity $t_1(x) = c \cdot x$, so that:

$$dW_1 = \frac{x\ n(x)\ dx}{\int_0^{x_{max}} x\ n(x)\ dx} = \frac{x\ n(x)\ dx}{M_{1,n}} \qquad (7)$$

Integrals of the type $\int_0^\infty x^k n(x)\,dx$ are designated as the k-th moment of the n-distribution and represented by the symbol $M_{k,n}$. Since the maximum granule size is x_{max} and hence $n(x > x_{max}) = 0$, we also have

$$\int_0^{x_{max}} x^k n(x)\,dx = M_{k,n} \tag{8}$$

It should be noted that equations 5-7 assume an even distribution of the orientation, which in general is not to be expected with a packing and must therefore be verified. A criterion for such a distribution can be found (Ref. 2).

The cross-sectional area is a function of the granule size, the distance ζ between the centre of gravity and the section, the orientation ϕ and Θ and also the granule form parameters $\varepsilon_i(x)$; $f = f(x, \zeta, \phi, \Theta, \varepsilon_i(x))$. For x = const. there is a region B(x) in the (ζ, ϕ, Θ) space whose inner points correspond to sections. From the function $f(x, \zeta, \phi, \Theta, \varepsilon_i(x))$ it is possible to determine the sub-region dB(x, f, df), which results in cross-sections of the size f...f + df. This sub-region will in general not be simply correlated, this depending on the shape of the granules. The desired probability dW_2 can be calculated from

$$dW_2 = dB(x, f, df) \; / \; B(x) \tag{9}$$

So far as we are aware, dW_2 has hitherto been calculated only for spheres. Even with rotation ellipsoids the expressions become awkward.

Equations 5-7 and 9 provide a general method of determining the desired magnitudes dW_1 and dW_2; x_u can in principle be found from $f(x, \zeta, \phi, \Theta, \varepsilon_i)$. The problem is thus solved.

3. Special case of spheres (Wicksell)

For spherical granules dW_1 is calculated using equation 7, and the section surface is given by

$$f = (\pi/4)\ (x^2 - 4\zeta^2) \tag{10}$$

The region B(x) extends from $-(x/2) \leq \zeta \leq +(x/2)$, and from equation 10 dB(x, f, df) is given by

$$dB = 2(1/\pi)\ (x^2 - (4f/\pi))^{-1/2} df \tag{11}$$

The factor 2 allows for the fact that dB consists of two equally large sub-regions at $-\zeta$ and $+\zeta$.

Hence dW_2 is given by

$$dW_2 = (2/\pi x)\ (x^2 - (4f/\pi))^{-1/2} df \tag{12}$$

The lower integration limit x_u follows from equation 10, putting $\zeta = 0$:

$$x_u = (4f/\pi)^{1/2}$$

Hence $z^*(f)$ and $z(\Theta)$ can be written respectively as follows:

$$z^*(f)\ df = \frac{2\ df}{\pi M_{1,n}} \int_{x_u}^{x_{max}} n(x)\ (x^2 - (4f/\pi))^{-1/2}\ dx \tag{13}$$

$$z(\Theta)\ d\Theta = \frac{\Theta\, d\Theta}{M_{1,n}} \int_{0}^{x_{max}} n(x)\ (x^2 - \Theta^2)^{-1/2}\ dx \tag{14}$$

Equation 14 was already given in 1925 by Wicksell (see Ref. 3).

The distribution densities of the f and Θ respectively have a singularity at $x = \Theta$. It can be proved by **transition** to the distribution sums that this does not disturb the normalization. After circumscribing the integration regions the following expression is obtained for $Z(\Theta) = \int_0^\Theta z(t)\, dt$ (see Ref. 2):

$$Z(\Theta) = 1 - \left[\frac{1}{M_{1,n}} \int_{\Theta}^{x_{max}} (x^2 - \Theta^2)^{1/2}\, n(x)\, dx \right] \qquad (15)$$

The relations between the moments of the n(x) and $z(\Theta)$ distributions are ascertainable (see Ref. 1) and can be expressed as follows:

$$M_{2k,z} = \frac{M_{2k+1,n}}{M_{1,n}} \quad \frac{2.4.6....(2k)}{3.5.7....(2k+1)}$$

$$k = 1,2, \qquad (16)$$

$$M_{2k-1,z} = \frac{M_{2k,n}}{M_{1,n}} \quad \frac{1.3.5....(2k-1)}{2.4.6....(2k)} \cdot \frac{\pi}{2}$$

$$M_{2k,n} = \frac{M_{2k-1,z}}{M_{-1,z}} \quad \frac{2.4.6....(2k)}{1.3.5....(2k-1)}$$

$$k = 1,2, \qquad (17)$$

$$M_{2k+1,n} = \frac{M_{2k,z}}{M_{-1,z}} \quad \frac{3.5.7....(2k+1)}{2.4.6....(2k)} \cdot \frac{\pi}{2}$$

The mean section f_m and the mean diameter Θ_m can immediately written down as follows:

$$\theta_m = M_{1,z} = \frac{\pi}{4} \frac{M_{2,n}}{M_{1,n}} \qquad (18)$$

$$f_m = \frac{\pi}{4} M_{2,z} = \frac{\pi}{6} \frac{M_{3,n}}{M_{1,n}} \qquad (19)$$

4. Section through a regular arrangement

The typical phenomena associated with a section through a regular arrangement will be considered on a plane model. Fig. 5 shows a square packing of circular discs. In the centre of a disc let a system of coordinates be set up and subdivided so that the centres of all the circles are lattice points. There are two types of intersection line, distinguishable according as to whether the series of section lengths along a line of intersection are repeated periodically or not. This distinction applies both to sections which pass through centres and to those which do not. It applies both to finite and to infinite packings. In the latter case periodic sequences arise only when the tangent of the angles of intersection is rational.

Irrational tangent values lead to sections without periodicity. These cases have a larger probability, since the irrational numbers possess a greater power than the rational.

With the periodic sequences there arise only a finite number of different section lengths and hence a discrete distribution. The number of section lengths is dependent upon the indexing of the lines of intersection. The higher the indexing, the greater the number of section lengths.

The region of the section lengths is increasingly occupied as the indexing increases. It can be proved that the mean section length here approximates to the value for the uniform random arrangement. If no periodicity occurs even with the infinitely extended packing, then any value of the section length is possible, the distribution and mean value again corresponding to those of the uniform random arrangement (see Ref. 2).

When going from the plane model to the three-dimensional packing, it should be noted that the inclination of the section surfaces is determined by two angles. This considerably reduces the possibilities of obtaining discrete distributions of the Θ values, so that in general the distributions and their mean values are as in the uniform random distribution. In view of these considerations the agreement of the measured distribution with the theoretical cannot be used as a sufficient test for distinguishing between a regular arrangement and a uniform random arrangement. This applies also to the distinction between the general and the uniform random arrangement, since the former will provide just as many distributions in the regions of equal granule size distributions as the uniform random arrangement.

5. Conclusions

In principle it is possible to calculate the distribution of the section surfaces. Owing to the difficulties of computation only the solutions for packings of spheres have hitherto been determined. It can be seen subjectively from the pattern of the section surfaces whether the arrangement is regular. If it is not, then the Wicksell equation does not enable us to distinguish between a general or a uniform random arrangement. However, the correlation between the moments makes it possible to test the mixture condition with regard to granule size distribution, since the moments of the z and n distributions clearly depend upon one another. The statement about the mixture condition gathers weight the more moments are used for comparison purposes. For many problems it is very important to avoid separation according to granule size.

It can be proved that the mean of the surface porosity ε_F is equal to the mean of the volume porosity ε_V. The homogeneity of the packing in ε_V can be tested at various points from the measurement results of the ε_F. When a medium is present in the void with a density not very different from that of the granules, ε_F can be determined with greater accuracy than ε_V.

Lastly, the variance from ε_F up to an arbitrary constant can be estimated from a model by considering the packing as a mixture of solid granules and "void particles" (see Ref. 2). For a uniform random distribution the relative variance must be inversely proportional to the size of the random sample.

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Fig.1 Section through a packing of lead spheres 0.67 to 1.08 mm in diameter with a regular arrangement

Fig.2 Section through a packing of lead spheres 0.97 to 1.08 mm in diameter with an irregular arrangement[1]

Fig.3 Section through a packing of limestone granules, size 0.1 to 1.1 mm, with an irregular arrangement[1]

Fig.4 Irregularly shaped granule. Sketch to illustrate the position parameter

Fig.5 Two-dimensional model of a regular arrangement of spheres