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CALCULATION OF THE ABSORPTION AND FISSION RESONANCE INTEGRALS OF U-235 BY A FITTING PROCEDURE

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SUMMARY

A fitting procedure has been set-up to evaluate quickly the resonance integrals of fissile nuclides in multigroup neutron calculation models.

For this purpose, the results of resonance integrals calculated by the refined numerical Nordheim method (ZUT and TUZ codes) are fitted into a formula based on the classical resonance theory, thus determining for each nuclide and type of event a set of parameters (three) dealing with the energy group considered.

The U-235 values are reported for the energy group scheme of the lattice code PINOCCHIO.

KEYWORDS

Introduction

Numerical methods for resonance integral calculations have been largely developed (ref. 1-3) and compared with success to the experimental results. The use of these methods is nevertheless for cell reactivity calculations extremely limited by the important machine time required. As a matter of example, a ZUT-TUZ calculation for the U-238 resonance integral takes of the order of 5 minutes of execution time on the IBM-7090 which is a prohibitive time in comparison with the times needed for lattice cell design calculation codes.

In order to bypass this difficulty in the frame of the development of the PINOCCHIO cell reactivity code, a fitting procedure has been set-up to correlate the results of ZUT and TUZ calculations by means of the analytical expression derived from the theory of resonance integrals.

1. Derivation of the analytical formula

In homogeneous systems the resonance integral of an absorber can be generally written as:

$$I = \int \frac{\Sigma_p \, \sigma_a(u)}{N_a \, \sigma_t(u)} \, du \qquad (1)$$

where:

$\dfrac{\Sigma_p}{N_a}$ is the potential scattering cross-section per atom of absorber.

$\sigma_a(u)$, $\sigma_t(u)$ are the lethargy dependent absorption and total cross-sections in the neighbourhood of the resonance. This formula can be evaluated after introduction of the Breit-Wigner equations and integration over the Energy Range into:

$$I = \frac{\frac{1}{2} \pi \sigma_p \frac{\Gamma_\gamma}{E_0}}{\sqrt{\frac{\sigma_p}{\sigma_0} \left(1 + \frac{\sigma_p}{\sigma_0} - \gamma \frac{1}{\sigma_p}\right)}} \qquad (2)$$

with:

Γ_n the neutron (scattering) width of the resonance.

Γ_γ the absorption width of the resonance.

E_0 the energy of the resonance centre.

σ_p the potential macroscopic in-scattering cross-sections of atoms which are able to contribute to a flat slowing down density spectrum into the resonance.

σ_0 the total cross-sections at the resonance centre.

$\Gamma = \Gamma_n + \Gamma_\gamma$

$$\gamma = g_j \, \sigma_{pa} \frac{\Gamma_n}{\Gamma}$$

and:

σ_{pa} the potential absorber cross-section.

g_j the statistical spin factor.

This formula has been evaluated assuming that:

1) the resonance cross-section can be expressed as a function of the energy by Breit-Wigner's equations;

2) the resonance is situated sufficiently far from the neutron sources to guarantee that a flat slowing-down density in the resonance range can be assumed;

3) The lethargy increase per collision for the moderating atoms in consideration is sufficiently large compared with the resonance width (NR) to make more than one collision in the resonance very improbable. This means that the slowing-down density spectrum in the resonance range is very insensitive to the removal of neutrons so that again the source spectrum can be considered to be flat.

4) The resonance is sufficiently narrow that in the expressions
the dependence on the energy of the maximum cross-section, re-
action width and energy can be neglected.
5) The temperature is so low, that no Doppler broadening needs
be assumed.
Moreover it has been shown by different authors, that only
these atoms will have to be counted, which in reality are able
to contribute to a flat slowing down density spectrum in the
whole resonance region. Thus in the narrow resonance case, when
the resonance is considered to be small with respect to the
slowing down of energy of scattering nuclei, any of them has
to be counted.

In the case of the NRIM, where only the absorber itself is
considered not   able to scatter into the resonance regions,
indeed this contribution is not counted. It is evident that
these cases are only the limiting ones and that more generally,
the potential scattering may be formulated as:

$$\sigma_p = \sum_i \lambda_i \, \sigma_{pi}$$

where σ_{pi} is the potential macroscopic scattering cross-section
per atom of absorber.
λ_i is the efficiency of the scatterer to contribute to a flat
collision density spectrum in the resonance region. It may
be shown that under certain conditions this value can be ex-
pressed as a function of the resonance practical width and the
atomic mass of the scatterer in question.

If the resonance absorber is not homogeneously dispersed in
the scattering medium (rod in moderator) it may be shown that
another term has to be added to the potential cross-section,
taking into account the heterogeneity effect. This term, ac-

cording to Wigner rational approximation for the rod escape probability can be written as:

$$\sigma_{het} = \frac{1}{\bar{l}\ N_a}$$

where:
$\bar{l}$ is the mean chord length;
N_a the absorber atom density (cm^{-3}).
Recalling:

$$N_a = \frac{\rho_F\ A\ \varepsilon}{\mathcal{M}_F} \qquad and \qquad \bar{l} = 2\ R_{eff}$$

where:
ρ_F is the density of the fuel
A Avogadro's constant
ε the enrichment
$\mathcal{M}_F$ the fuel molecular weight
R_{eff} the effective radius of the fuel rod when a cluster is considered (or the physical radius in the case of a single rod), one finds easily:

$$\sigma_{het} = \frac{\mathcal{M}_F}{4\ A\ \varepsilon}\ \frac{S_{eff}}{M}$$

where:
S_{eff} is the effective surface of the clustered fuel (cm^2/cm)
M the mass of fuel per unit of height (gr/cm).
A very useful formula for clusters is (ref. 5) :

$$\frac{Seff}{M} = \frac{S}{M} \left(\frac{1 - D}{1 + 0.1D} \right)$$

where D is Dancoff correction factor.

Summarising, the resonance integral for a single resonance in heterogeneous systems can be written in the following form:

$$I = \frac{C \, \sigma_p^*}{\sqrt{\dfrac{\sigma_p^*}{\sigma_o} \left(1 + \dfrac{\sigma_p^*}{\sigma_o} - \dfrac{\gamma}{\sigma_p^*} \right)}} \qquad (3)$$

where

$$C \equiv \frac{1}{2} \pi \frac{\Gamma_\gamma}{E_o} \qquad \text{and} \qquad \gamma \equiv \frac{g_j \, \Gamma_n \, \sigma_{pa}}{\Gamma}$$

By putting $C' = C \sqrt{\sigma_o}$; $\sigma_p^* = \sigma_p + \sigma_{het}$ and $\delta = (\sigma_p^*)^{-1}$ one finds easily that:

$$I = \frac{C'}{\sqrt{\dfrac{1}{\sigma_o} + \dfrac{1}{\sigma_p^*} - \dfrac{\gamma}{\sigma_p^{*2}}}} \qquad (4)$$

or:

$$I^{-2} = \frac{1}{C'^2} \left(\frac{1}{\sigma_o} + \delta - \gamma \delta^2 \right) \qquad (5)$$

or:

$$I^{-2} = a_0 + a_1 \delta + a_2 \delta^2 \tag{6}$$

with: $\quad a_0 = \dfrac{1}{\sigma_0 C'^2} \quad ; \quad a_1 = \dfrac{1}{C'^2} \quad ; \quad a_2 = -\gamma a_1$

and: $\quad \delta = \dfrac{1}{\sigma_p^*} = \left(\displaystyle\sum_i \lambda_i \sigma_{pi} + \dfrac{\mathcal{M}_F}{4 A \varepsilon} \dfrac{S_{eff}}{M} \right)^{-1}$

The formula (6) has been taken as start-up point for the fitting procedure. It has been assumed that this expression, determined for a single resonance, would be valid for all the resonances belonging to each of the energy groups considered.
The coefficients a_0, a_1 and a_2 have been determined as follows: a set of calculations with the Nordheim approach ZUT-TUZ have been carried out by varying the input parameters involved in the definition óf (δ), here considered as the indépendent variable of the problem. The Nordheim approach can be considered today as one of the most refined numerical methods for resonance integral calculations. Starting from these results a fitting procedure has been set-up which determines the a_i values as follows.

3. The fitting program

This is based on the condition that the sum of the squares of the relative errors $\left(\dfrac{I_Z - I_F}{I_Z} \right)$ between the fitted (I_F) and the reference data (I_Z), is a minimum.

This gives rise to the equations:

$$S_n \left(\frac{I_F - I_z}{I_z} \right)^2 = \text{minimum}$$

or:

$$S_n \left[1 - \left(\frac{a_0 + a_1 \delta + a_2 \delta^2}{I_z} \right) \right]^2 = \text{minimum}$$

After differentiating to a_i one obtains the equations:

$$Y_j = \underset{i}{S} \, a_{i-1} X_{ij} \qquad\qquad (\text{i and j} = 1 \longrightarrow 3)$$

with:

$$Y_j = \frac{1}{I_z} S_n \, \delta^{j-1}$$

$$X_{ij} = \frac{1}{I_z^2} S_n \, \delta^{i+j-2}$$

where Sn is the sum over the number of values to be fitted. This set of linear equations can easily be solved by determinant methods.

3. Application to the U-235 Resonances

The calculations of the resonance integrals were performed by changing the following parameters:
a) Temperature
b) Energy group

c) enrichment range (see next)

d) chemical compound of U-235 U-238

e) reaction type (fission, absorption)

Two problems arose however by fitting the results:

1) the efficiency factors λ_i for atoms i(25, 28, O and C)
were not known "a priori". Therefore the root mean square va-
lues of the relative errors (standard deviations) were calculated
for a number of λ_i values. It turned out, that the standard
deviation was very sensitive to the U-235 and U-238 efficiencies
(which were assumed to be equal). The influence of the Oxigen
and Carbon efficiencies was much less.

2) Another problem was caused by the large range of the va-
riables δ and I, which resulted in a low sensitivity of the
a_i values to the extremes of the δ and I scales.
Therefore the enrichment dimension was split-up into two over-
lapping parts; the high enrichment range from 100 to 4.98%
and the low enrichment one from 13.5% to natural. For each of
them a set of constants has been evaluated.

To cover the whole range of U-235 enrichments with a rela-
tively small number of reference points and besides this, to
decrease the mesh at the relatively more important enrichment
range between natural and 10% calculations were chosen with
enrichments of e^{-n} with n ranging between 0 and 5.
Natural enrichment corresponds to n = 4.945. The effective
U-235 atom fractions (with respect to the Uranium mixture
235 + 238) were then the following:

n	Enrichment	n	Enrichment
0	100%	3	4.98%
1	36.79%	4	1.83%
2	13.53%	5	.673%

To be able to fit the constants in the analytical formula (3)
calculations were made for rod radii of 0.5, 1.5 and 2.5 cm
radius. The shadowing effect in the fuel pins as a result
of clustering was taken into account by introducing two diffe-
rent Dancoff correction factors (0; representing the unsha-
dowed case, and 0.6 for rods in very compact clusters). The
densities of the METAL, OXIDE and CARBIDE used in the calcu-
lations were 19.13; 10.11 and 12.55 respectively.

The potential cross-sections used in the ZUT calculations
were respectively:

Description	σ_P (barns)
Uranium-235	12.5
Uranium-238	10.
Oxigen	3.
Carbon	4.65

Because of the resonance Doppler broadening three temperatures
were chosen (20°, 820°, 1620°C resp.).
The statistical weight factor for U-235 was: .5.
Finally the resonance parameters used in the ZUT calculations
are given in Table I. These resonances were collected into 3
groups as indicated in the following Table:

Group No.	E max.	E min.
5	36. eV	29. eV
6	29. eV	11. eV
7	11. eV	2.3 eV

4. Results

As an example is given in Fig. 1 the plot of the standard
deviations as a function of the efficiency factors for U-235
and U-238 for low enrichments at 293°K . It is clearly seen

that a minimum exists. Fig. 1 shows furthermore that there is
a strong dependence of the optimal values on the group number
(or medium energy). For higher energies the efficiency, al-
though remaining small, increases. Moreover, it should be
noted that the value of the minimum error (s.d.) decreases
with energy. This is evidently due to the better adoptability
of the analytical theory (smaller resonances, smaller Doppler
broadening, energy dependence etc.). Figs. 2 and 3 give examples
of plots of calculated and fitted points for the "adapted" case
with the minimum s.d. and a less optimised case. For other tem-
peratures and enrichment ranges the plots have a similar form.
Tables II and III give the λ_i values and their standard devia-
tions of the four nuclei for the high and low enrichment ranges
in the fit for the absorption integral. Because the λ_i values
which cause a minimum of the s.d. depend primarily on the to-
tal width of the resonances these values are the same for the
absorption as well as the fission case.

The standard deviations themselves depend on the gamma/
total width ratio, and need not be the same for the absorption
and fission integrals. The absorption integrals are calculated
as formerly described and needed no changement of the original
ZUT program. The so found values of a_i are shown for the dif-
ferent temperatures, enrichment ranges and chemical compounds
in Tables IV to IX. With respect to the fission integrals the
difficulty appeared that these values could not be directly
calculated because the total width Γ_t could not be entered
separately.

Insertion of Γ_f instead of Γ_γ does not calculate properly
the depletion of slowing down density in the resonance and

thus gives erroneous results. Therefore the calculation was
done manually, by multiplying the absorption integral by the
ratio of fission to absorption width. However, because of the
large number of integrals (about 2000 per compound per tem-
perature), this work has only been performed for one tempe-
rature (293°K). The resulting values were again fitted as
formerly to obtain the respective a_i fission values. A check
has been made for group 7 at 1893°K. It turned out, as might
be foreseen, that these values were a practical constant frac-
tion of the absorption ones, about independent from the chemi-
cal compound and fuel temperature as shown in Table X for the
high enrichment range. For this reason the calculation of the
fission a_i values for temperatures 1093 and 1893°K was made
by multiplying the absorption ones by the ratios obtained
at 293°K.

In the meantime, however, the ZUT program has been adapted,
which will enable us to recalculate rigorously these data.
The values so obtained for the fission integrals are shown
in Tables XI to XVI.

In the Tables the coefficients whose absolute value is less
than one are written as $\pm$ X.XXXX - Y to signify $\pm$ X.XXXX $\cdot$ 10^{-Y}.

FIG. 1 : Standard Deviation between the Calculated and the Fitted Values for Metal at 293 °K as a Function of λ_U

FIG. 2 : Fit of the Calculated Points of Group 7A for Metal at 293°K with the Curve $\frac{1}{RI^2} = a_o + a_1\delta + a_2\delta^2$ ($\lambda_U = 0$)

FIG. 3 : Fit of the Calculated Points of Group 7A for Metal at 293°K with the Curve $\frac{1}{RI^2} = a_o + a_1\delta + a_2\delta^2$ ($\lambda_U = -0.1$)

U 235 (%)

○	13.53
▲	4.98
+	1.83
●	0.67

from ZUT Calculations

——— Fitted Curve
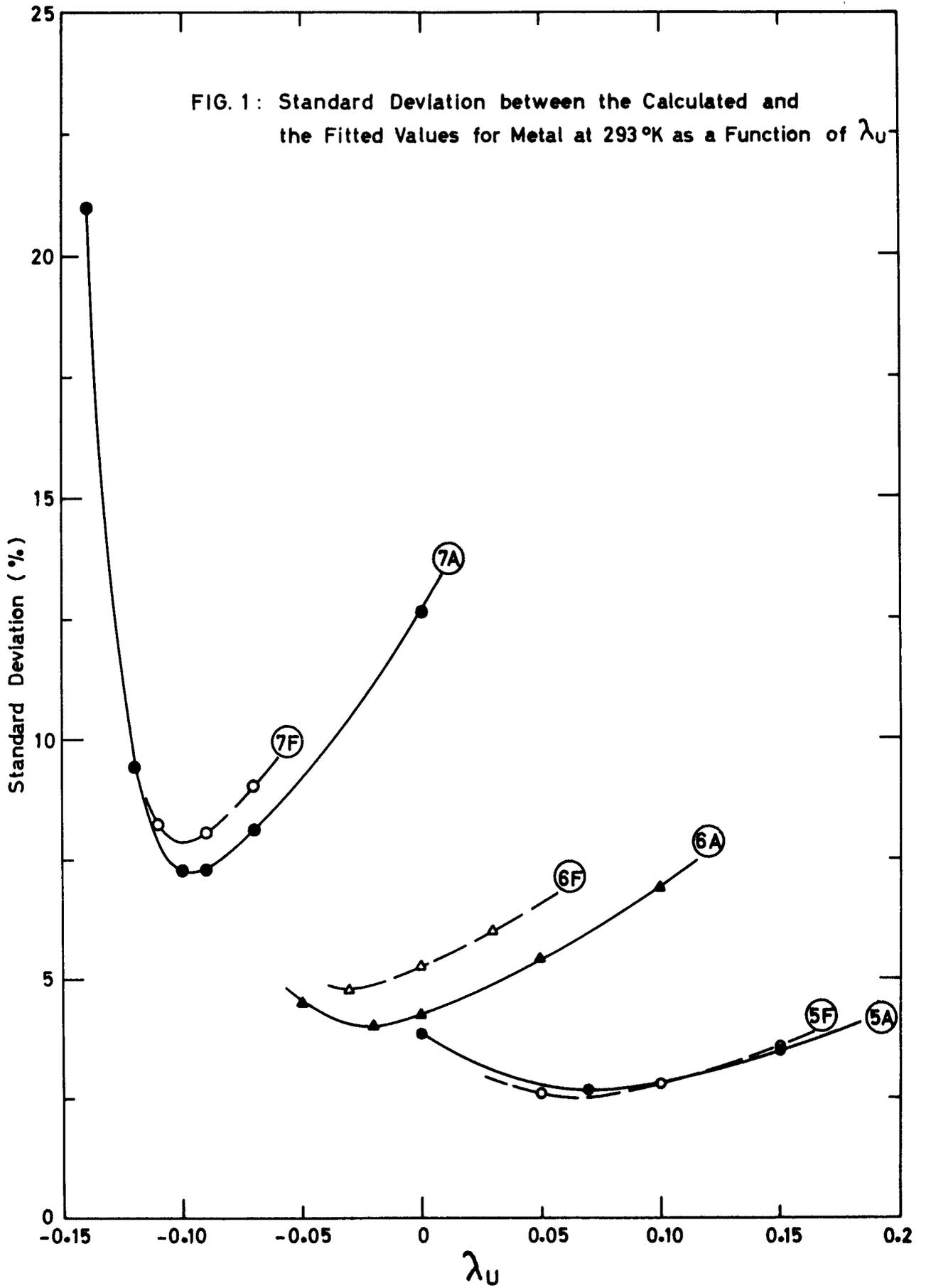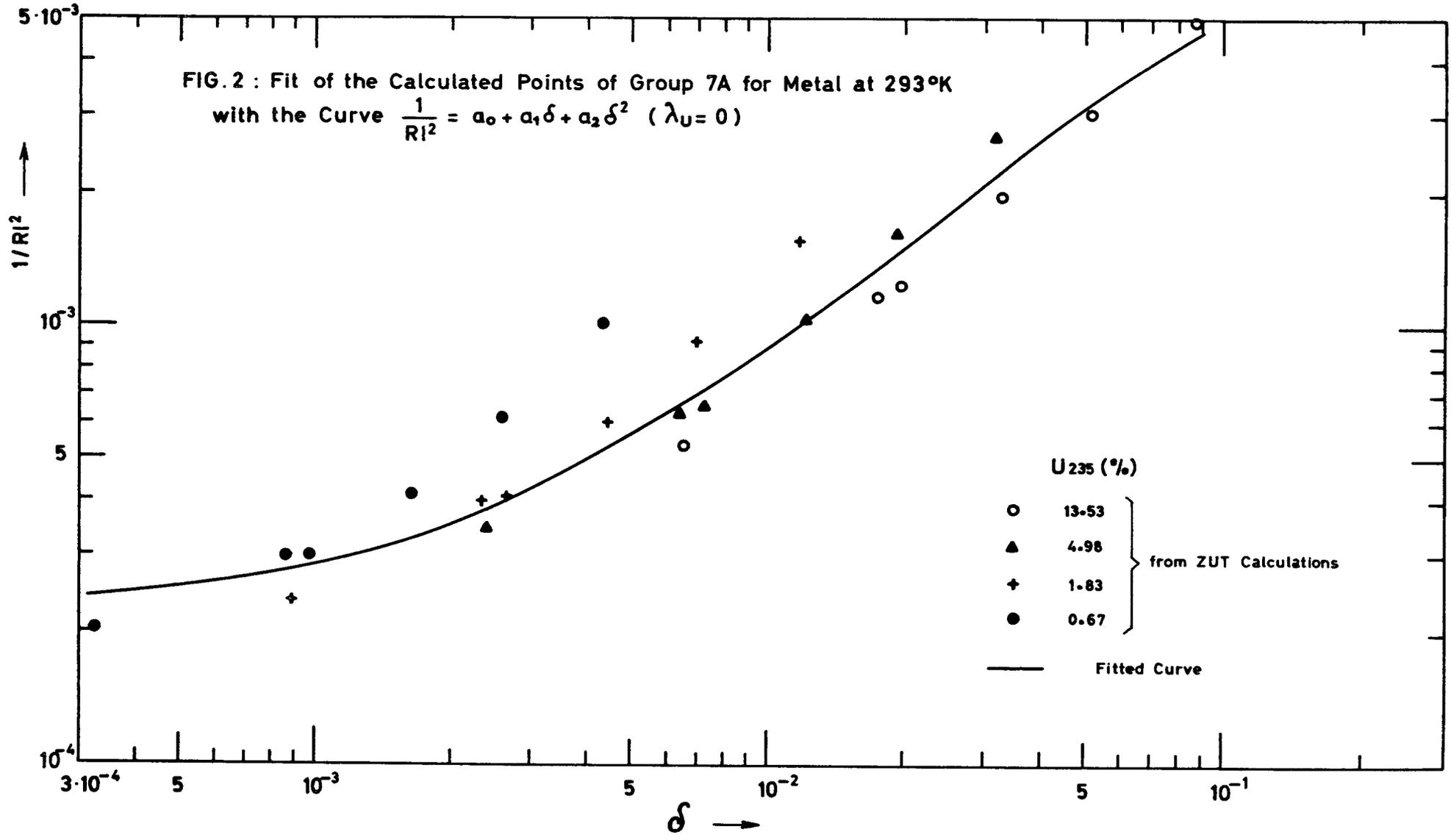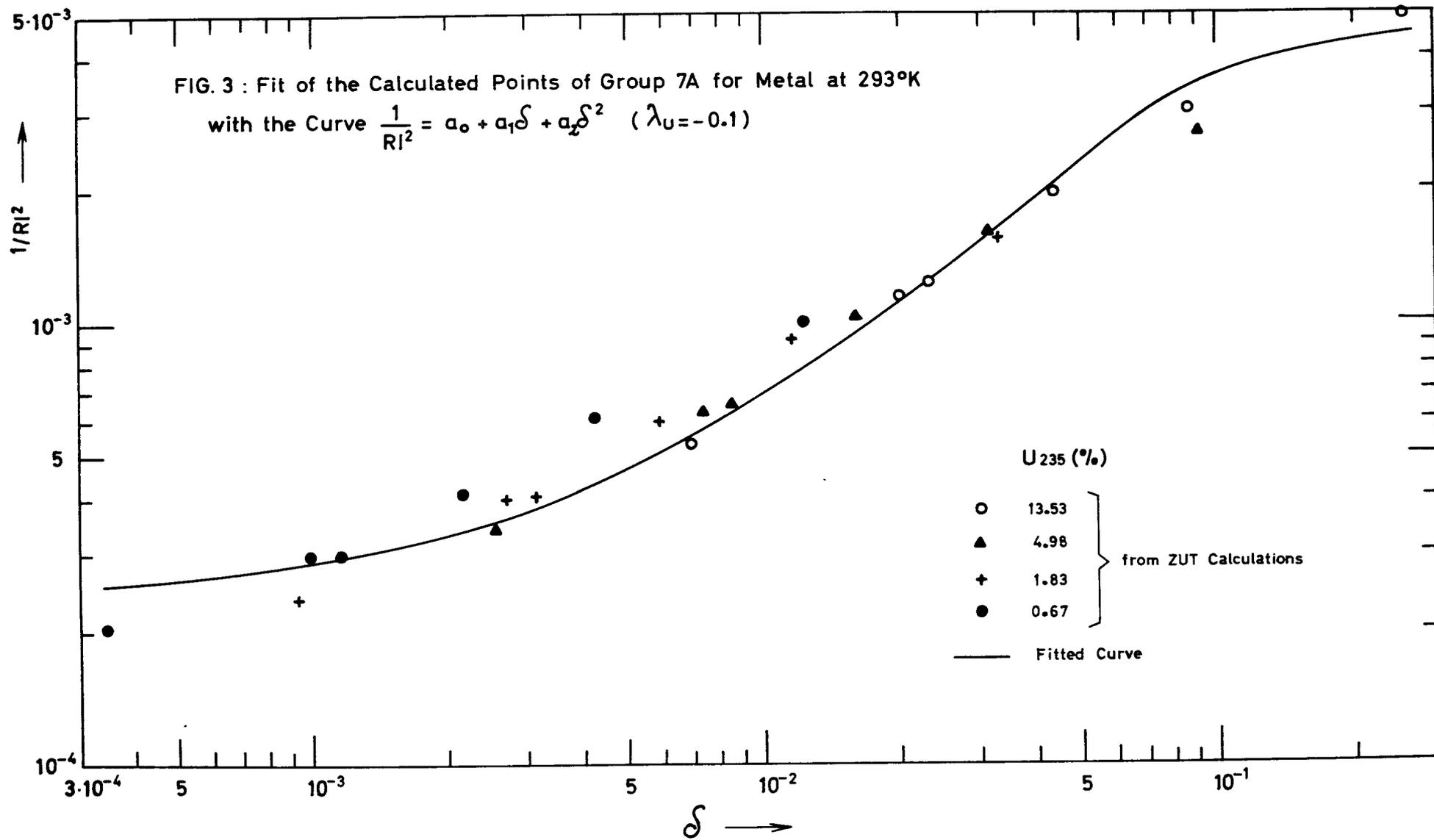
Table I

U^{235} Resonance Parameters

No.	E(eV)	$\Gamma n(10^{-3}eV)$	$\Gamma f(10^{-3}eV)$	$\Gamma \gamma(10^{-3}eV)$
4	2.89	.005	100	140
5	3.15	.023	100	137
6	3.61	.047	45	87
7	4.85	.056	3.8	40.8
8	5.48	.022	330	370
9	5.84	.012	150	195
10	6.20	.042	130	170
11	6.40	.260	10	45
12	7.095	.120	28	64
13	8.795	1.18	82	132
14	9.28	.20	95	160
15	9.74	.04	140	180
16	10.18	.06	58	95
17	11.65	.63	4	40
18	12.39	1.29	24	68
19	12.86	.04	60	83
20	13.28	.04	51	91
21	13.71	.044	70	110
22	14.00	.42	400	440
23	14.53	.115	23	61
24	15.415	.250	49	98
25	16.095	.350	17	52
26	16.68	.265	86	132
27	18.05	.39	90	160
28	18.97	.10	25	65
29	19.29	3.1	52	102
30	20.13	.13	65	120
31	20.60	.23	33	92
32	21.06	1.58	21	68
33	22.93	.45	42	92
34	23.41	.69	8	40
35	23.61	.80	85	165
36	24.27	.33	52	92
37	25.62	.44	150	190
38	26.49	.43	150	190
39	27.05	.10	200	240
40	27.79	.76	60	123
41	28.34	.16	96	140
42	29.63	.18	22	67
43	30.59	.21	77	150
44	30.85	.49	17	59
45	32.06	1.95	38	90
46	33.51	1.80	20	60
47	34.38	1.80	40	90
48	34.81	1.05	49	109
49	35.19	4.30	120	183

Table II

Fitted efficiencies (λ) and standard deviations (Δ) for the high enrichment range

	293°K			1093°K			1893°K		
	λ_U	λ_O	λ_C	λ_U	λ_O	λ_C	λ_U	λ_O	λ_C
5.A	.20	.74	1.3	.12	1.1	1.47	.10	1.13	1.51
6.A	.10	.95	.95	.05	1.01	1.05	.04	1.03	1.08
7.A	-.05	.17	.30	-.05	.20	.30	-.05	0.13	.31

Δ (%)

	Δ_{MET}	Δ_{OX}	Δ_{CAR}	Δ_{MET}	Δ_{OX}	Δ_{CAR}	Δ_{MET}	Δ_{OX}	Δ_{CAR}
5.A	2.82	2.51	2.85	3.15	2.17	2.64	3.75	2.04	2.47
6.A	6.49	2.30	4.09	7.18	2.46	4.41	7.71	2.34	4.21
7.A	8.47	4.71	3.33	8.86	4.74	3.02	9.10	4.74	3.03

Table III

Fitted efficiencies (λ) and standard deviations (Δ) for the low enrichment range

	293°K			1093°K			1893°K		
	λ_U	λ_O	λ_C	λ_U	λ_O	λ_C	λ_U	λ_O	λ_C
5.A	.1	.87	1.24	.02	1.24	1.69	0	1.2	1.73
6.A	0	.8	.9	-.04	1.07	1.14	-.06	1.16	1.29
7.A	-.09	.15	.22	-.1	.15	.2	-.1	.15	.15

Δ (%)

	Δ_{MET}	Δ_{OX}	Δ_{CAR}	Δ_{MET}	Δ_{OX}	Δ_{CAR}	Δ_{MET}	Δ_{OX}	Δ_{CAR}
5.A	.9	2.53	2.25	3.14	.68	1.34	3.46	.60	1.28
6.A	4.25	3.56	6.33	4.59	.64	2.68	4.90	.58	2.26
7.A	7.31	3.34	5.38	7.81	3.75	5.58	8.18	4.33	5.68

Table IV

Absorption a_i values of the different energy groups for high

enrichments in different compounds at

TEMPERATURE 293°K

	a_0	a_1	a_2

Group 5.A

	a_0	a_1	a_2
Metal	2.2486 – 3	9.0267 – 1	– 2.3025
Oxide	2.3355 – 3	9.3464 – 1	– 3.3661
Carbide	1.9992 – 3	8.0081 – 1	– 4.388 – 1

Group 6.A

	a_0	a_1	a_2
Metal	2.691 – 4	4.6541 – 2	– 9.4619 – 2
Oxide	1.8302 – 4	4.9127 – 2	– 1.3668 – 1
Carbide	1.6967 – 4	4.3563 – 2	– 9.2331 – 1

Group 7.A

	a_0	a_1	a_2
Metal	3.5367 – 4	3.4511 – 2	– 2.3214 – 2
Oxide	2.4581 – 4	3.6645 – 2	– 8.1318 – 2
Carbide	2.3737 – 4	3.8338 – 2	– 6.9001 – 2

Table V

Absorption a_i values of the different energy groups for high

enrichments in different compounds at

TEMPERATURE 1093°K

	a_0	a_1	a_2

Group 5.A

	a_0	a_1	a_2
Metal	2.4567 - 3	5.9578 - 1	- 1.2206
Oxide	2.3023 - 3	5.4524 - 1	4.1958 - 1
Carbide	2.0527 - 3	5.6190 - 1	2.3852 - 1

Group 6.A

	a_0	a_1	a_2
Metal	2.5709 - 4	3.3327 - 2	- 5.4859 - 2
Oxide	1.6807 - 4	3.6722 - 2	- 9.0129 - 2
Carbide	1.5766 - 4	3.2473 - 2	- 6.4646 - 2

Group 7.A

	a_0	a_1	a_2
Metal	3.0400 - 4	3.1564 - 2	- 2.1175 - 2
Oxide	2.2790 - 4	3.3303 - 2	- 7.3747 - 2
Carbide	2.2158 - 4	3.4043 - 2	- 5.9561 - 2

Table VI

Absorption a_i values of the different energy groups for high

enrichments in different compounds at

TEMPERATURE 1893°K

	a_0	a_1	a_2
Group 5.A			
Metal	2.5628 - 3	4.8682 - 1	- 9.3862 - 1
Oxide	2.3095 - 3	4.4840 - 1	+ 4.9720 - 1
Carbide	2.0977 - 3	4.6275 - 1	+ 3.1670 - 1
Group 6.A			
Metal	2.4936 - 4	2.8449 - 2	- 4.4545 - 2
Oxide	1.6323 - 4	3.1595 - 2	- 6.9737 - 2
Carbide	1.5357 - 4	2.8112 - 2	- 5.2067 - 2
Group 7.A			
Metal	2.8883 - 4	2.9312 - 2	- 1.9557 - 2
Oxide	2.2525 - 4	2.8679 - 2	- 5.8116 - 2
Carbide	2.1510 - 4	3.1799 - 2	- 5.5527 - 2

Table VII

Absorption a_i values of the different energy groups for low

enrichments in different compounds at

TEMPERATURE 293°K

	a_0	a_1	a_2

Group 5.A

	a_0	a_1	a_2
Metal	2.5569 - 3	7.9073 - 1	- 2.9609
Oxide	2.4160 - 3	9.5511 - 1	- 6.6181
Carbide	2.3714 - 3	6.1825 - 1	+ 2.0726

Group 6.A

	a_0	a_1	a_2
Metal	1.8665 - 4	5.6680 - 2	- 3.2979 - 1
Oxide	1.7260 - 4	5.7897 - 2	- 1.6589
Carbide	1.8934 - 4	4.1517 - 2	- 5.8504 - 1

Group 7.A

	a_0	a_1	a_2
Metal	2.3241 - 4	5.0572 - 2	- 1.4203 - 1
Oxide	1.9216 - 4	5.4516 - 2	- 7.2574 - 1
Carbide	2.1072 - 4	4.4638 - 2	- 3.4419 - 1

Table VIII

Absorption a_i values of the different energy groups for low
enrichments in different compounds at

TEMPERATURE 1093°K

	a_0	a_1	a_2

Group 5.A

	a_0	a_1	a_2
Metal	2.5954 − 3	5.3014 − 1	− 2.2583
Oxide	2.3093 − 3	5.7356 − 1	− 7.1684
Carbide	2.3677 − 3	4.5202 − 1	+ 4.8226

Group 6.A

	a_0	a_1	a_2
Metal	1.8520 − 4	4.1137 − 2	− 1.9544 − 1
Oxide	1.4821 − 4	4.6778 − 2	− 1.228
Carbide	1.5381 − 4	3.3495 − 2	− 4.0623 − 1

Group 7.A

	a_0	a_1	a_2
Metal	2.2867 − 4	4.1759 − 2	− 1.0208 − 1
Oxide	1.9069 − 4	1.6239 − 2	− 5.8055 − 1
Carbide	2.0803 − 4	3.7109 − 2	− 2.6318 − 1

Table IX

Absorption a_i values of the different energy groups for low
enrichments in different compounds at

TEMPERATURE 1893°K

	a_0	a_1	a_2

Group 5.A

	a_0	a_1	a_2
Metal	2.6112 – 3	4.4704 – 1	– 1.9099
Oxide	2.3148 – 3	4.6466 – 1	– 6.3100
Carbide	2.3638 – 3	3.7388 – 1	+ 3.8386

Group 6.A

	a_0	a_1	a_2
Metal	1.8650 – 4	3.3697 – 2	– 1.4014 – 1
Oxide	1.4815 – 4	4.0261 – 2	– 1.0131
Carbide	1.5274 – 4	2.9300 – 2	– 3.1344 – 1

Group 7.A

	a_0	a_1	a_2
Metal	2.2483 – 4	3.8908 – 2	– 9.3686 – 2
Oxide	1.9038 – 4	4.2560 – 2	– 5.1388 – 1
Carbide	2.0746 – 4	3.2749 – 2	– 2.1856 – 1

Table X

Ratios α_i of a_i values for fission and absorption for group 7

(2.38 - 11.eV) for 293 and 1893°K

	α_0	α_1	α_2
TEMPERATURE 293°K			
Metal	3.6452	3.2850	3.0523
Oxide	3.8765	3.2043	3.1496
Carbide	3.9475	3.1968	3.0801
TEMPERATURE 1893°K			
Metal	3.7970	3.4750	3.1951
Oxide	3.8767	3.4238	3.3894
Carbide	3.9454	3.4023	3.2798

The fission a_i values for 1093°K and 1893°K have been calculated in this way by multiplication of the absorption ones with the ratios at 293°K.
(Table XI- XVI).

Table XI

Fission a_i values of the different energy groups for high enrichments in different compounds at

TEMPERATURE 293°K

	a_0	a_1	a_2

Group 5.F

	a_0	a_1	a_2
Metal	9.4762 - 3	4.1453	- 10.846
Oxide	9.7996 - 3	4.2902	- 16.253
Carbide	8.3725 - 3	3.6292	- 2.5012

Group 6.F

	a_0	a_1	a_2
Metal	1.4375 - 3	1.7699 - 1	- 3.4558 - 1
Oxide	1.0179 - 3	1.9187 - 1	- 5.7734 - 1
Carbide	9.5980 - 4	1.7295 - 1	- 3.9841 - 1

Group 7.F

	a_0	a_1	a_2
Metal	1.2892 - 3	1.1337 - 1	- 7.0857 - 2
Oxide	9.5288 - 4	1.1742 - 1	- 2.5612 - 1
Carbide	9.3704 - 4	1.2256 - 1	- 2.1253 - 1

Table XII

Fission a_i values of the different energy groups for high

enrichments in different compounds at

TEMPERATURE 1093°K

	a_0	a_1	a_2

Group 5.F

	a_0	a_1	a_2
Metal	1.0353 − 2	2.7357	− 5.7497
Oxide	9.6603 − 3	2.5028	+ 2.0259
Carbide	8.5965 − 3	2.5465	+ 1.3595

Group 6.F

	a_0	a_1	a_2
Metal	1.3732 − 3	1.2674 − 1	− 2.0036 − 1
Oxide	9.3475 − 4	1.4342 − 1	− 3.8071 − 1
Carbide	8.9186 − 4	1.2892 − 1	− 2.7895 − 1

Group 7.F

	a_0	a_1	a_2
Metal	1.1081 − 3	1.0369 − 1	− 6.4633 − 2
Oxide	8.8345 − 4	1.0671 − 1	− 2.3227 − 1
Carbide	8.7471 − 4	1.0883 − 1	− 1.8345 − 1

Table XIII

Fission a_i values of the different energy groups for high

enrichments in different compounds at

TEMPERATURE 1893°K

	a_0	a_1	a_2

Group 5.F

	a_0	a_1	a_2
Metal	1.0800 − 2	2.2354	− 4.4214
Oxide	9.6905 − 3	2.0582	+ 2.4007
Carbide	8.7850 − 3	2.0971	+ 1.8051

Group 6.F

	a_0	a_1	a_2
Metal	1.3319 − 3	1.0819 − 1	− 1.6269 − 1
Oxide	9.0783 − 4	1.2340 − 1	− 2.9457 − 1
Carbide	8.6872 − 4	1.1161 − 1	− 2.2467 − 1

Group 7.F

	a_0	a_1	a_2
Metal	1.05?8 − 3	9.6291 − 2	− 5.9695 − 2
Oxide	8.7318 − 4	9.1895 − 2	− 1.8304 − 1
Carbide	8.4913 − 4	1.0166 − 1	− 1.7103 − 1

Table XIV

Fission a_i values of the different energy groups for low

enrichments in different compounds at

TEMPERATURE 293°K

	a_0	a_1	a_2

Group 5.F

Metal	1.0733 - 2	3.6531	- 12.514
Oxide	1.0205 - 2	4.3443	- 25.356
Carbide	1.0047 - 2	2.7953	+ 9.8039

Group 6.F

Metal	1.0369 - 3	2.2937 - 1	- 1.3674
Oxide	9.3298 - 4	2.4666 - 1	- 7.5486
Carbide	1.0134 - 3	1.7536 - 1	- 2.6688

Group 7.F

Metal	8.7813 - 4	1.7103 - 1	- 4.7881 - 1
Oxide	7.2189 - 4	1.9191 - 1	- 2.6669
Carbide	7.8955 - 4	1.5741 - 1	- 1.3106

Table XV

Fission a_i values of the different energy groups for low

enrichments in different compounds at

TEMPERATURE 1093°K

	a_0	a_1	a_2

Group 5.F

	a_0	a_1	a_2
Metal	1.0895 – 2	2.4492	– 9.5445
Oxide	9.7543 – 3	2.6088	– 27.464
Carbide	1.0031 – 2	2.0437	+ 22.812

Group 6.F

	a_0	a_1	a_2
Metal	1.0288 – 3	1.6647 – 1	– 8.1035 – 1
Oxide	8.0114 – 4	1.9929 – 1	– 5.5878
Carbide	8.2323 – 4	1.4148 – 1	– 1.8531

Group 7.F

	a_0	a_1	a_2
Metal	8.6400 – 4	1.4122 – 1	– 3.4413 – 1
Oxide	7.1636 – 4	1.6277 – 1	– 2.1334
Carbide	7.7947 – 4	1.3086 – 1	– 1.0021

Table XVI

Fission a_i values of the different energy groups for low

enrichments in different compounds at

TEMPERATURE 1893°K

	a_0	a_1	a_2

Group 5.F

	a_0	a_1	a_2
Metal	1.0961 − 2	2.0653	− 8.0720
Oxide	9.7775 − 3	2.1135	− 24.176
Carbide	1.0015 − 2	1.6904	+ 18.157

Group 6.F

	a_0	a_1	a_2
Metal	1.0361 − 3	1.3636 − 1	− 5.8106 − 1
Oxide	8.0084 − 4	1.7152 − 1	− 4.6100
Carbide	8.1751 − 4	1.2376 − 1	− 1.4298

Group 7.F

	a_0	a_1	a_2
Metal	8.4949 − 4	1.3158 − 1	− 3.1583 − 1
Oxide	7.1520 − 4	1.4982 − 1	− 1.8884
Carbide	7.7733 − 4	1.1548 − 1	− 8.3223 − 1

Acknowledgment

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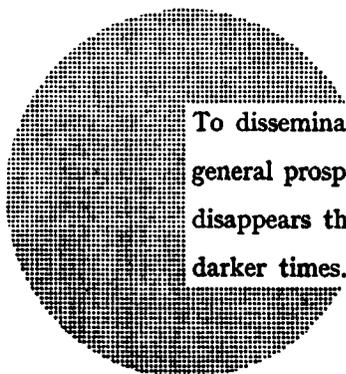5. LEVINE: Resonance integral calculations for U-238 lattice, Nucl. Sc. and Engn. 16(1963) 271-279

To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

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